 Complex numbers

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| 2.1 Arithmetic of complex numbers and solving quadratic equations.  The student is able to:  Appreciate the necessity of introducing the symbol, where, in order to solve quadratic equations.  Students could be introduced to as a device by which quadratic equations with real coefficients could be always solvable.  Write down the real partand the imaginary partof a complex number.  Add, subtract and multiply complex numbers written in the form  Find the complex conjugateof the number    Divide a complex numberby a complex number  Write down the condition forandto be equal.  Prove that there are always two square roots of a non-zero complex number.  The arithmetic and algebra of complex numbers would then be developed and eventually it could be shown that there exist 2 complex roots for a complex number.  Find the square roots of a complex number  In finding the square roots of,the statement , where,,, andare real, leads to the need to solve the equations and  Examining graphs of these curves for various values of and will lead to the conclusion that two roots will always exist for a complex number.  Solve quadratic equations of the form, where, and are complex.  This then leads to the discovery that a quadratic equation with complex coefficients will have 2 complex roots. | * Introduce the concept ofas a means by which any quadratic equation has solutions – this then leads to the proof that any non-zero complex number has two square roots. * Explicitly link theandto the plotting of a complex number on an Argand diagram. * Define addition, subtraction and multiplication of complex numbers algebraically but reinforce the concepts of addition and subtraction on the Argand diagram. * Develop an understanding of the complex conjugate of a complex number – link explicitly to Geometrical representation on Argand diagram. * Link the equality of complex numbers to work in Preliminary Mathematics on equivalence of quadratics. * Techniques for finding the square roots of complex numberwhere,,, and are real link to the graphs ofand leading to the conclusion that two roots will always exist. | [John & Betty’s Journey into Complex Numbers](http://mathforum.org/mbower/johnandbetty/)  [Eddie Woos Complex Playlist (Youtube)](https://www.youtube.com/watch?v=gHUHZXjpwOE)  [Complex Numbers by Topic 1995 - 2006 (Web)](http://members.optuszoo.com.au/hscsupport/95%20to%2006%20Ext%202%20Complex%20Numbers.pdf)  [Tips and Tricks for finding complex square roots (p. 8 & 9) (Web](https://www.maths.unsw.edu.au/sites/default/files/hsc_tips_and_tricks-handbook.pdf))  [Wikibook Complex Numbers (Web)](https://en.wikibooks.org/wiki/HSC_Extension_1_and_2_Mathematics/4-Unit/Complex_numbers)  Arithmetic of Complex Numbers [[Keynote](https://drive.google.com/open?id=0B2rBnOj-8kBAV1FtaDFHSEVXZms) | [ppt](https://drive.google.com/open?id=0B2rBnOj-8kBAOGVackoxYnpDWEE) | [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBAOGVackoxYnpDWEE)]  Summary Notes Complex Numbers [[Word](https://drive.google.com/open?id=0B2rBnOj-8kBANUEyNm1ZUXJDUnc) | pdf]  A Complex Numbers Assignment [[Word](https://drive.google.com/open?id=0B2rBnOj-8kBAOHdZTzVDbENuM2c) | [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBAOHdZTzVDbENuM2c)]  A Complex Numbers Assignment Solutions ([Word](https://drive.google.com/open?id=0B2rBnOj-8kBANTNOVGJ6WlZZNm8))  [Polynomials with Complex Variables (pdf)](https://drive.google.com/open?id=0B2rBnOj-8kBAMWxiZW1RaG8xTDg)  [Complex Numbers - Dimensions (Youtube)](https://www.youtube.com/watch?v=2kbM96Jr4nk&feature=youtu.be) |
| 2.2 Geometric representation of a complex number as a point  The student is able to:  Appreciate that there exists a one to one correspondence between the complex number.  and the ordered pair  Plot the point corresponding toon an Argand diagram.  Define the modulusand argumentof a complex number.  The geometrical meaning of modulus and argument forshould be given and the following definitions used:    is any value offor whichand . Whileis commonly assigned a value betweenand, the general definition is needed in order that the relations involvinggiven below are valid.  Find the modulus and argument of a complex number.  Writein modulus-argument form.  Prove basic relations involving modulus and argument  Students should at least be able to prove the following relations:                  Use modulus-argument relations to do calculations involving complex numbers.  Recognise the geometrical relationships between the point representingand points representing,(c real) and .  The fact that multiplication by corresponds to an anticlockwise rotation throughabout O, thatis the reflection ofin the real axis and that multiplication by c corresponds to an enlargement about O by a factor c, should be used on simple geometrical exercises. | * Reinforce the concept that all skills developed on the Cartesian Plane apply equally to the Argand diagram. * Write complex number in modulus argument form – students need to be able translate backwards and forwards readily. * Right angled Trigonometry – often the easiest way to find the modulus or the argument. * We are concerned with the Principal Argument of a complex number   Develop a geometrical understanding that multiplication byrepresents arotation in an anticlockwise direction | Question 4. (iii) (1987)   * Let OABC be a square on an Argand diagram where O is the origin. The points A and C represent the complex numbers z and iz respectively. Find the complex number represented by B. * The square is now rotated about O through 45° in an anticlockwise direction to OA'B'C'. Find the complex numbers represented by the points A', B' and C'   New South Wales Board of Studies (1989), Mathematics 4 Unit Year 12 Syllabus, p27.  [Introduction to Complex Numbers - NRich](http://nrich.maths.org/9859/note)  [Plotting Complex Numbers on the Argand Diagram (pdf)](https://drive.google.com/open?id=0B2rBnOj-8kBAMWxiZW1RaG8xTDg) |
| 2.3  Geometrical representations of a complex number as a vector.  The student is able to:  Appreciate that a complex number z can be represented as a vector on an Argand diagram.  Familiarity with the vector representation of a complex number is extremely useful when work on curves and loci is encountered.  Appreciate the geometrical significance of the addition of two complex numbers.  Given the points representingandfind the position of the point representing, where    Appreciate that the vector representingcorresponds to the diagonal of a parallelogram with vectors representingandas adjacent sides.  Given vectorsand, construct vectorsand  Students need to be able to interpret the expression as the magnitude of a vector joiningto the point representing.  Students need to recognise that the expressionrefers to the angle, which a vector joining the point representing to the point representing, makes with the positive direction of the real axis.  Givenand, construct the vector  prove geometrically that | * Demonstrate geometrically why the product of a conjugate pair of complex numbers is always real. * Demonstrate the product of two complex numbers as a vector product on an Argand Diagram * Make explicit links to the vector arithmetic undertaken in Physics * Vector arithmetic can be undertaken in Geogebra to show the geometrical implications | [Complex Numbers in Geogebra](https://wiki.geogebra.org/en/Complex_Numbers).  [What is a vector? (Youtube)](https://youtu.be/127MpSs0ZkY) |
| 2.4  Powers and roots of complex numbers  The student is able to:  Prove, by induction, that  for positive integers  Prove thatfor negative integers  for negative integersfollows algebraically from the previous result.  Find any integer power of a given complex number.  Find the complex nth roots ofin modulus-argument form.  Sketch the nth roots ofon an Argand diagram  Students should realise that points corresponding to the nth roots of are equally spaced around the unit circle with centre O and so form the vertices of a regular n-sided polygon.  Illustrate the geometrical relationship connecting the nth roots of . | * Some students may not have done induction at this stage – may be necessary to come back to proof. * Reinforce the nth roots are evenly spaced on the unit circle. * Student must be able to plot any integer power of a complex number on the Argand diagram. | [Coroneos Cube Roots with Solutions (pdf)](https://drive.google.com/open?id=0B2rBnOj-8kBAOGVackoxYnpDWEE)  [Some notes of Complex Roots of Unity (pdf)](https://drive.google.com/open?id=0B2rBnOj-8kBAWV9RQmx3SE84UHM) |
| 2.5 Curves and Regions  The student is able to:  Given equations,  (,real), sketch lines parallel to the appropriate axis  Given an equation, sketch the corresponding line.  Given equations , sketch the corresponding circles.  Given equations ,,sketch the corresponding rays.  Sketch regions associated with any of the above curves (eg the region corresponding to those satisfying the inequality  Typical curves and regions are those defined by simple equations or inequalities such as              Give a geometrical description of any such curves or regions.  Sketch and describe geometrically the intersection and/or union of such regions.  Simple intersections, such as the region common toand, and corresponding unions, need be done.  Sketch and give a geometrical description of other simple curves and regions.  Examples need only involve replacingbyin relations such as,,. They need not include discussion of curves such as, wherelies on a unit circle. | * Student must be familiar with all common locus types and be able to recognise them * Both geometrical and algebraic approaches should be used * Discussions should include: lines parallel to the axes (e.g. ,); perpendicular bisector; Circles; Rays; Major Arcs * Most locus problems can be solved by substitutingand obtaining a recognisable Cartesian equation – particularly in more complex loci such as | [(YouTube)](https://www.youtube.com/watch?v=x-HBMht3NV8)  [(Youtube)](https://www.youtube.com/watch?v=pYs3lA0qNMM&index=13&list=PL0YzbD4vPtxI_qdvhe4s9-3aseA1I2LLr)  [(Youtube)](https://www.youtube.com/watch?v=ar_x9Tww-F0)  Complex Locus 1 [ [Keynote](https://drive.google.com/open?id=0B2rBnOj-8kBAWjdnOEV1MTRJZUE) | [ppt](https://drive.google.com/open?id=0B2rBnOj-8kBAblpSbEhUMmk0QUU) | [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBAUmJpNm5scnNHMkU) ]  Complex Locus 2 [ [Keynote](https://drive.google.com/open?id=0B2rBnOj-8kBAblpSbEhUMmk0QUU) | [ppt](https://drive.google.com/open?id=0B2rBnOj-8kBAblpSbEhUMmk0QUU) | [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBAblpSbEhUMmk0QUU) ]  [Summary of Complex Locus (pdf)](https://drive.google.com/open?id=0B2rBnOj-8kBAMWxiZW1RaG8xTDg) |