 Year 11 Mathematics Standard - Rhombus

Unit title: Applications of Measurement (MS-M1)

Duration: 2 weeks (rhombus course)

Rationale: Students develop knowledge of the concepts of measurement and demonstrate fluency with its application.

Topic focus: The principal focus of this subtopic is to appreciate inherent error in measurements and to become competent in solving practical problems involving energy, mass, perimeter, area, volume and capacity.

Prior knowledge required

* MA4-12MG calculates the perimeters of plane shapes and the circumferences of circles
* MA4-13MG uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area
* MA5.1-8MG calculates the areas of composite shapes, and the surface areas of rectangular and triangular prisms
* MA5.1-9MG interprets very small and very large units of measurement, uses scientific notation, and rounds to significant figures

Language considerations

Area

* Teachers should reinforce with students the use of the term 'perpendicular height', rather than simply 'height', when referring to this attribute of a triangle. Students should also benefit from drawing and labelling a triangle when given a description of its features in words.
* Students may improve their understanding and retention of the area formulas by expressing them in different ways, for example, 'The area of a trapezium is half the perpendicular height multiplied by the sum of the lengths of the parallel sides', 'The area of a trapezium is half the product of the perpendicular height and the sum of the lengths of the parallel sides'.
* The use of the term 'respectively' in measurement word problems should be modelled and the importance of the order of the words explained, eg in the sentence 'The perpendicular height and base of a triangle are 5 metres and 8 metres, respectively', the first attribute (perpendicular height) mentioned refers to the first measurement (5 metres), and so on.
* The abbreviation m2 is read as 'square metre(s)' and not 'metre(s) squared' or 'metre(s) square'. Similarly, the abbreviation cm2 is read as 'square centimetre(s)' and not 'centimetre(s) squared' or 'centimetre(s) square'.
* When units are not provided in an area question, students should record the area in 'square units'.
* Standard form is being used instead of scientific notation (scientific notation is still being used in the K-10 Syllabus)

Outcomes

A student

* MS11-3 solves problems involving quantity measurement, including accuracy and the choice of relevant units
* MS11-4 performs calculations in relation to two-dimensional figures
* MS11-9 uses appropriate technology to investigate, organise and interpret information in a range of contexts
* MS11-10 justifies a response to a given problem using appropriate mathematical terminology and/or calculations

Assessment (including formative and summative)

Formative assessment:

* Student response cards for instant feedback.

For example: [Student Response Cards](http://www.theteachertoolkit.com/index.php/tool/student-response-cards) http://www.theteachertoolkit.com/index.php/tool/student-response-cards

* [Plickers](https://www.plickers.com/): https://www.plickers.com/ or [Kahoot](https://kahoot.com/?utm_name=controller_app&utm_source=controller&utm_campaign=controller_app&utm_medium=link)%20for%20progress%20checks): https://kahoot.com/?utm\_name=controller\_app&utm\_source=controller&utm\_campaign=controller\_app&utm\_medium=link) for progress checks

| Outcomes and content | Teaching and learning strategies and evidence of learning | Resources |
| --- | --- | --- |
| M1.1 Practicalities of measurement | N/A | N/A |
| * Review the use of different metric units of measurement including units of area, take measurements, and calculate conversions between common units of measurement, for example kilometres to metres or litres to millilitres (ACMEM090, ACMEM092) ◊
 | Language:Students should have a good understanding of the commonly used metric system units, symbols and prefixes (see link).Teaching Strategies:* List the different units of measurement that students are familiar with (add to it if necessary but students should know mm, cm, m, km, mL, L, kL, mg, g, Kg, Tonnes)
* Discuss the need to use appropriate units of measure in the real world (see Activity 1 from the Mathematics Developmental Continuum P-10 link)

Activities:* Converting between different attributes - provide students with a couple of different scenarios to research and provide their answer with justification
	+ For example - In a game show, you win a prize. You are offered 1 kg of $1 coins, or 1 square metre covered in 20c pieces. Which would you choose?
* Aluminium Foil Prank - show students the photo of the foil print and then pose the question: How much would it cost to cover the whole classroom in aluminium foil?
* Measuring Stations - set up a series of stations (tables) where students are provided with the task of measuring length, area, mass and volume at each station. Students are provided with a 2-column worksheet where one is labelled “guess” and the other is labelled “actual size”. This is a hands on activity to get students to familiarise themselves with the metric system and appropriate sizes.

Evidence of Learning:Students can explain how they use appropriate measuring instruments and the metric system to measure the length, area, mass and/or volume of given objects.Adjustments:Depending on the activity, if students were asked to measure objects, the degree of difficulty can be modified based on the object provided, for example asking students to measure the surface area of a door instead of a bottle of water. | [Commonly used metric system units, symbols and prefixes](http://www.us-metric.org/commonly-used-metric-system-units-symbols-and-prefixes/):http://www.us-metric.org/commonly-used-metric-system-units-symbols-and-prefixes/[Activity 1 and 2 - Converting between different attributes](http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/convertmeas40.aspx#a1) (from Mathematics Developmental Continuum p 10):http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/continuum/Pages/convertmeas40.aspx#a1[Practical Measuring Activities](http://www.mathematicshed.com/uploads/1/2/5/7/12572836/practicalmeasuring.pdf):http://www.mathematicshed.com/uploads/1/2/5/7/12572836/practicalmeasuring.pdf[Foil Activity](http://robertkaplinsky.com/work/foil-prank/):http://robertkaplinsky.com/work/foil-prank/Hands on Activity - Measuring Stations:[Teaching the metric system](http://www.us-metric.org/tips-to-educators-for-teaching-the-metric-system-and-ideas-for-schools-celebrating-national-metric-week/): http://www.us-metric.org/tips-to-educators-for-teaching-the-metric-system-and-ideas-for-schools-celebrating-national-metric-week/ |
| * Calculate the absolute error of a reported measurement and state the corresponding limits of accuracy ◊
	+ Investigate types of errors, for example, human error or device limitations
	+ Calculate the percentage error of a reported measurement
 | Language:Error in measurement is the difference between the measured value and the actual value. Two people can measure the same thing but get two different measurements for different reasons.Teaching Strategies:Research with students types of errors that can occur in a measured value:* Systematic error - this occurs to the same extent in each measurement of a series of measurements. For example when the needle of a bathroom scale is not correctly adjusted to read zero when no weight is present (also known as device or measurement tool limitations)
* Random error - this occurs in any measurement as a result of variations in the measurement technique. Such errors may include:
	+ Parallax error - when a person views the scale of a measuring instrument at an angle rather than from directly in front of it (note that if this is done consistently then it becomes a systematic error)
	+ For example - a person sitting in the passenger seat of a car may glance at the speedometer and think the driver is going above the speed limit when in fact the driver sitting in front of the speedometer can see that the speed of the car is right on the speed limit.
	+ Limit of reading - the smallest unit on a measuring instrument can limit the accuracy of the reading, that is, if the wrong instrument was used then the measurement obtained would have an error

Ensure students understand the difference between absolute error and relative error.* Absolute error - actual size of the error expressed in the appropriate units
* Relative error - absolute error expressed as a fraction of the actual measured quantity. They can also be expressed as percentage errors
	+ For example - if the length of a table was measured as 1.2m long and the error found to be 0.1. The absolute error is 0.1m and the relative error is 0.1/1.2 (or 8.3% (aka percentage error))

Activities:Limits of Accuracy; Discovering the limits - students use a non-standard method of measurement, for example, one student lying on the floor as a “Tyler”. They use this measurement/person to measure the length of the room and find that it is 3.2 “Tylers”, however because you can’t measure with half a person, the room becomes 3 “Tylers”, as you can’t measure with something that you don’t know, there is no marking that shows .2 of a “Tyler”* They then measure the hall which they find is 14.7 “Tylers”, which this time is estimated up to 15 “Tylers”
* Ask students question such as:
	+ What do we know about the length of the hallway?

(They know that it is MORE than 14.5 “Tylers” because otherwise they would have rounded down.)* + Could it be 15.4 “Tylers” and still be measured as 15?
	+ Could it be 15.5?
* Advance the questions to say: If “Tyler” could accurately be portioned into tenths, how much more accurate could your measurement be?
* If I told you the hallway was 11m long, what could you tell me about that?
* Emphasise the following to students - We cannot be completely accurate when measuring anything, we can only say that something is closer to 6 “Tylers” than it is to 5 or 7 “Tylers”

See attached PDF titled “Accuracy of Measurements Teacher Notes”Design a perfect ruler (limit of reading) - Students have to measure a pencil using three different rulers. They have to create 3 different rulers with different calibrations (calibrations 1mm apart, 1cm apart, 5cm apart) and then come to the conclusion about the best ruler and why? Then they test those rulers by measuring their hand span. The video ‘Limits of reading’ can be used by teachers or students for an explanation of limits of reading.Percentage Error - Choose a convenient object available to you that can reasonably be measured in metres, centimetres and/or millimetres, for example a desk or whiteboard.* Have students measure the object with 3 separate measuring instruments. For example: a metre ruler, a 30cm ruler and a tape measure.
* For each measurement instrument, have students calculate the limits of accuracy.
* Using the measurements taken with each instrument, calculate the percentage error.
* Have students evaluate/assess the different things they have discovered: What instrument gave the most accurate measurement? What instrument has the smallest/biggest limits of accuracy? What instrument gave the smallest percentage measurement and why? How does the percentage error relate to/impact the instrument and limits of accuracy?

Evidence of Learning:Limits of Accuracy/Percentage Error* Students understand that the limits of accuracy are  the smallest units of measurement and can demonstrate this in the context of common units of measurement, such as in mm, cm, m, and km.
* Students understand how limits of accuracy relate to percentage error and how accurate the measurement is.

Adjustments:* Limits of Accuracy
	+ Teachers can be much more structured in how they ask questions/facilitate discussion when looking at errors of measurement. 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6 then asking students which whole number each decimal to (0, 1, or 2) so students see the exact point at which the limits change () and which value a measurement is rounded to.
* Percentage Error
	+ Extension: As well as having three different measuring instruments, use three different students with the same instruments. For example, do three different people end with three different measurements using the metre ruler
	+ This brings in a connection with types of error, in this case human error, and how this affects the measurements being made.
 | [Precision and Accuracy](https://www.researchgate.net/figure/23965199_fig1_Bull%27s-eye-representation-of-accuracy-and-precision-With-respect-to-the-Bland-Altman):https://www.researchgate.net/figure/23965199\_fig1\_Bull%27s-eye-representation-of-accuracy-and-precision-With-respect-to-the-Bland-Altman[Parallax Error](http://aphysicsteacher.blogspot.com.au/2009/12/what-is-parallax-error.html):A detailed explanation with diagrams on parallax error.http://aphysicsteacher.blogspot.com.au/2009/12/what-is-parallax-error.html(Teachers can use this to further their understanding of parallax error)[Percentage error](https://www.youtube.com/watch?v=9zZUGJEzPCk):Powerpoint presentation for calculating absolute and percentage error.https://www.youtube.com/watch?v=9zZUGJEzPCk[Accuracy of Measurements Teachers Notes](http://archive.teachfind.com/ttv/static.teachers.tv/shared/files/13625.pdf):http://archive.teachfind.com/ttv/static.teachers.tv/shared/files/13625.pdf[Limits of reading](https://www.youtube.com/watch?v=bAuuMr5pCwk) (it demonstrates the concept of limit of reading):https://www.youtube.com/watch?v=bAuuMr5pCwk |
| * Use standard form and standard metric prefixes in the context of measurement, with and without a required number of significant figures ◊
 | Language:Students should have a good understanding of the commonly used metric system units, symbols and prefixes (see link)Teaching Strategies:* Teacher to have a discussion with students on the importance of using significant figures. Answers can include:
	+ To make sure that our answer does not appear to be more precise than it actually is
	+ This is important when working with measurement as there are varying degrees of certainty and precision
* Teachers can show this video to students for a visual demonstration of significant figures - Why are significant figures important? (see YouTube video)
* Explain the difference between decimal places and significant figures
* Explain the basic rules of significant figures
	+ Any non-zero digit is significant.
	+ Zeros between non-zero digits are always significant. Example: 3,606 has 4 significant figures.
	+ Zeros that indicate the decimal point are not significant. Example: 360,600 has 4 significant figures.
	+ Zeros following a decimal are significant. Example: 3.60 has 3 significant figures but 3.6 has 2.
	+ Zeros appearing before a non-zero digit are not significant. Example: 0.009 only has 1 significant figure.

Activities:* Racing for significant figures - this activity is good for getting students to work in small groups and create friendly competition to assess students’ understanding but to also give students a chance to help each other gain that understanding of significant figures. (see link for further details - scroll down to where it says Significant figures race)
* Significant figures dominoes - an activity where students are given the dominoes (already cut out) and they have to link it based on the idea of estimating or rounding numbers to one significant figure.
* Create a quiz - Students read the rules for writing numbers to a given significant figure and create a quiz (5 questions) with answers
* Gridlocks - Students place the numbers in appropriate boxes matching to the corresponding significant figures. Students round off given 4 numbers to 1, 2, 3 and 4 significant figures and write them in respective boxes.

Evidence of Learning:* Students demonstrate their understanding of significant figures by completing the worksheet which includes a connecting cards task where students are provided with cards and they have to connect the cards based on the question and answer provided on the card. (See link)
* Quiz created by the students demonstrates their understanding of significant figures
* Correct entries in the gridlocks can be used to assess their learning

Adjustments:* Degree of difficulty can be altered using different questions when asking students to provide answer to a certain number of significant figures. For example, a basic question would be to round 221.56 to 2 sig. fig. whereas to a more complex question would be to round 0.0052003 to 2 sig. fig.
* Numbers on the gridlocks could be changed to make the work simpler or extend it for higher ability students.
 | [Commonly used metric system units, symbols and prefixes](http://www.us-metric.org/commonly-used-metric-system-units-symbols-and-prefixes/):http://www.us-metric.org/commonly-used-metric-system-units-symbols-and-prefixes/[YouTube: Why are significant figures important?](https://www.youtube.com/watch?v=VAuslY-Uuf4):https://www.youtube.com/watch?v=VAuslY-Uuf4[Racing for Significant Figures](https://www.carolina.com/teacher-resources/Interactive/significant-figures-game/tr10849.tr):http://www.carolina.com/teacher-resources/Interactive/significant-figures-game/tr10849.tr[Dominoes](https://www.tes.com/teaching-resource/significant-figures-6293139):https://www.tes.com/teaching-resource/significant-figures-6293139(Note - teachers can use their Google Education account to register for free to access the resources)[Create a quiz](http://www.staff.vu.edu.au/mcaonline/units/numbers/numsig.html):http://www.staff.vu.edu.au/mcaonline/units/numbers/numsig.html[Gridlocks](http://www.rsc.org/learn-chemistry/resources/gridlocks/downloads/SignificantFigures.pdf):http://www.rsc.org/learn-chemistry/resources/gridlocks/downloads/SignificantFigures.pdf[Connecting cards - Significant figures worksheet](https://www.tes.com/teaching-resource/maths-significant-figures-worksheet-6266264):https://www.tes.com/teaching-resource/maths-significant-figures-worksheet-6266264(Note - teachers can use their Google Education account to register for free to access the resources) |
| M1.2: Perimeter, area and volume | N/A | N/A |
| * Review and extend how to solve practical problems requiring the calculation of perimeters and areas of triangles, rectangles, parallelograms, trapezia, circles, sectors of circles and composite shapes (ACMGM018) ◊
 | Language:* Students must understand and recognise the names and diagrams of simples shapes (such as squares, rectangles, triangles, circles, rhombus, kite, parallelograms and trapeziums)
* Recognise the difference between perimeter and area
* Identify different features of a circle (radius, diameter, circumference, arc length, sector, chord, minor and major segment)

Teaching Strategies:Pretest - assess what students remember about finding the perimeter and area of simple shapes (such as squares, rectangles, triangles, circles, rhombus, kite, parallelograms and trapeziums) and go through the formulas if necessary.Use paper cutouts of simple figures to explain and demonstrate how students can calculate the perimeter and area of composite shapes using their understanding of simple shapes.* Extension strategy - students can be asked to discuss and investigate scenarios such as:
	+ How many rectangles with integer dimensions can you make with a perimeter of 50m?
	+ How do the areas of these rectangles compare in relation to the length and width of the rectangles and explain which dimensions gives the greatest area?
	+ (Provide students with pipe cleaners or string to create a physical representation. Teachers can also use the build a bigger field link for inspiration for further extensions.)

Use Geogebra to demonstrate the concept of sectors of circles and why the angle is importantDemonstrate how to calculate the perimeter and area of sectors of circles - emphasise the idea of the arc length is a fraction of the circumference of the circle and the area of a sector is a fraction of the area of a circleActivities:* Work out who has the bigger slice of pizza - each student is provided with a slice of a pizza (can be real or a cut depending on your budget) and they are to use a protractor to work out the angle of the sector and use the formula to calculate the area of the sector.
* Provide students with a variety of composite shapes (can use Australian states) and ask students to describe how to separate each shape/state into simpler figures. Then use the figures to estimate the area of each composite shape/state.

Evidence of Learning:Student will demonstrate their understanding of perimeter and area through an exit card activity with the questions:* Draw a shape and a composite shape with a perimeter of 20cm
* Draw a shape and a composite shape with an area of 100cm2

Adjustments:* Provide students with examples of simple or complex composite shapes based on their level of understanding and needs
* Use different units of measurement to assess student’s understanding in the exit card activity
* May be more helpful to teach this session before the practicalities of measurement section
 | [Pretest on Simple Shapes](https://www.proprofs.com/quiz-school/story.php?title=area-compound-shapes):https://www.proprofs.com/quiz-school/story.php?title=area-compound-shapes[Build a Bigger Field](https://teacher.desmos.com/activitybuilder/custom/56e19b4183ba3908118725dd):https://teacher.desmos.com/activitybuilder/custom/56e19b4183ba3908118725dd[Area of Circle Derivation](https://www.youtube.com/watch?v=YokKp3pwVFc):https://www.youtube.com/watch?v=YokKp3pwVFcTeachers and students to watch[Geogebra - Sectors](https://www.geogebra.org/m/AjaYC7rj):https://www.geogebra.org/m/AjaYC7rj[Sector Area](https://teacher.desmos.com/activitybuilder/custom/58d92ba29623f50ba8d7f2af):https://teacher.desmos.com/activitybuilder/custom/58d92ba29623f50ba8d7f2af |
| * Review the use of Pythagoras’ theorem to solve problems involving right-angled triangles
 | Language:Students will need to understand that the hypotenuse is the longest side of a right-angled triangle and is opposite to the right-angle.Teaching Strategies:* Use the visual demonstration videos on the right on Pythagoras’ theorem to consolidate how the rule works
* Provide students with revision on the Pythagoras’ Theorem rule and with a pretest of simple questions to ensure students know how to use the rule to find unknown sides of a right-angled triangle
* Guide students through worded problems by using the following steps
1. If a diagram has not been provided, draw one to represent the worded problem
2. Identify whether the unknown side is a hypotenuse or a short side
3. Apply the formula:  if the unknown side is the hypotenuse and consider using the formula  if the hypotenuse is given and the unknown side is one of the short sides
4. Conclude the problem with a worded answer

(See link on sample word problems for sample questions)Activities:* Proving Pythagoras Theorem - Students are to investigate different methods of proving that the theorem works and gain a deeper understanding:
	+ Students use the template provided (free download from link to the right) to discover the proof of the theorem. The area of the larger square can be manipulated to create the area of the two smaller squares. The link also includes a powerpoint animation to visually demonstrate this for students.
* Practical Application - In order to understand the applications of Pythagoras Theorem, it will be important to define for students what parts of the problem relate to Pythagoras, for example, a TV or Computer size is given by the length of its diagonal. Possible questions to ask students are:
	+ If I have a cabinet with a 55cm x 40cm opening, what is the maximum size TV that will fit? (Finding the Hypotenuse)
	+ My ladder is 4.2m long and the base sits 1.3m away from the wall. Is my ladder long enough to reach the window that is 3.7m up the wall? (Proof Question)
	+ If a roof rafter is 10m in length and extends 7m over the horizontal ceiling beams. How high above the ceiling does the roof extend? (Finding the shorter side)

Evidence of Learning:* Students are able to apply Pythagoras’ Theorem to find the value of an unknown side (whether it be one of the short side or the hypotenuse)
* Students demonstrate their understanding by converting worded questions into a diagram and worked solutions with a worded conclusion

Adjustments:Investigating Pythagoras Theorem Extension: A more advanced method to investigate the theorem is using the video link opposite to create the Origami Proof of Pythagoras Theorem. In addition to the theorem investigation, this activity also requires students to understand the need for a perfect square and experiment in how to create one to start off with. | Pythagoras’ Theorem videos:[Pythagoras’ Theorem Water Demo](https://www.youtube.com/watch?v=CAkMUdeB06o):https://www.youtube.com/watch?v=CAkMUdeB06o[Visual Proof of Pythagoras’ Theorem](https://www.youtube.com/watch?v=KHJRDSP5I8E):https://www.youtube.com/watch?v=KHJRDSP5I8E[Sample of Word Problems](http://www.math-only-math.com/word-problems-on-pythagorean-theorem.html):http://www.math-only-math.com/word-problems-on-pythagorean-theorem.html[Proving Pythagoras’ Theorem Worksheet Instructions](https://www.tes.com/teaching-resource/cut-and-stick-discover-pythagoras-theorem-6150236):https://www.tes.com/teaching-resource/cut-and-stick-discover-pythagoras-theorem-6150236[Origami Proof of Pythagoras’ Theorem](https://www.youtube.com/watch?v=z6lL83wl31E):https://www.youtube.com/watch?v=z6lL83wl31E |

Reflection and evaluation