 Geometrical applications of differentiation

Resources

[Dynamic calculus](https://schoolsequella.det.nsw.edu.au/items/07c79e99-fecb-4621-ba0e-ffa8524bb0aa/1/)

This resource combines a collection of Geogebra interactive learning objects with material explaining the significance and history of calculus, including the 'calculus controversy'. Note – needs java).[Geogebra](http://www.geogebra.org) can be used extensively in this topic. Downloadable resource or use online.[AMSI website](http://www.amsi.org.au/ESA_Senior_Years/SeniorTopic3/3_md/SeniorTopic3c.html) – Supporting Australian Mathematics Project (a guide for teachers of year 11 and 12) has a good comprehensive overview of Geometry of the derivative

Scope and sequence note

As geometrical applications of differentiation relies on the tangent to a curve and the derivative of a function (prelim) and is a prerequisite for the majority of the HSC course, it needs to be completed early in the HSC year.

| Content | Teaching strategies and activities | Resources |
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| Significance of the sign of the derivative. | When looking at the meaning of the first derivative, time should be spent on informally looking at the sign of the first derivative and drawing $f(x) and f'(x) $on a set of corresponding axis allowing for the visual transfer of information. For this and other parts of this topic, the use of “everyday” scenarios (either real or contrived) are encouraged, as the students need to be able to interpret the scenario as it pertains to the derivative of a function, move between  and  and **describe** (geometrically) the curves involved - including the words increasing, decreasing, stationary.The use of graphing software (Geogebra, Geometer’s Sketchpad etc) is useful for to show the dynamic changes of the first derivative (tangent) and how it relates to the shape of the curve.  | See HSC questions 2014 Q14e |
| Stationary points on curves. | Use a variety of definitions and scenarios to explore stationary points on curves, horizontal gradient,  Distinguish between **stationary points** and **turning point, maxima** and **minima, local** and **global maximums** and **minimums.**Use of [Frayer](http://www.worksheetworks.com/miscellanea/graphic-organizers/frayer.html) and [Venn](http://www.worksheetworks.com/miscellanea/graphic-organizers/venn.html) diagrams may help with consolidating the similarities and differences of these conceptsThe criteria of a turning point is the change of the sign of  around the stationary point.Use of tables, graphically summarising  round the stationary point are useful.Eg         Gradient direction     Local MinimumCare should be taken that if the term point of inflexion is used at this point that students do not assume all points of inflexion are horizontal. | [Tasia jigsaw file on stationary points](http://www.mrbartonmaths.com/resources/Jigsaws/Tarsia/6.%20A-Level/C1/5.%20Differentiation/Stationary_Points.xjsw)[Maths is Fun – Maxima and Minima](https://www.mathsisfun.com/calculus/maxima-minima.html) |
| The second derivative. The notation  | If the Newton-Leibniz controversy has not been discussed previously in the course, this is a good point to discuss why there are so many different ways of writing differential calculus and that students should be familiar with at least  and dot notation when differentiating with respect to time | The Calculus Controversy Although not directly connected to the second derivative notation this short video goes some way to explain why there are a number of different forms of notation |
| Geometrical significance of the second derivative. | Compare and contrast the difference between  concave upward concave downward point of inflectionUse memory aids such as Up like a cup (for the shape of concave upward curve portion)Down like a frown (for the shape of concave downwards curve portion)The nature of a stationary point can be found from the concavity around the stationary point, however, care must be taken with. A number of curves of this type should be encountered to understand the need for caution. | [BBC Bitesize – Differentiation](http://www.bbc.co.uk/bitesize/higher/maths/calculus/) |
| The sketching of simple curves. | Sketches should involve where possible:* y-intercepts,
* x-intercepts,
* stationary points and
* points of inflexion
* behaviour at large x and large y

All these points should be labelled on the sketch.Although sketches do not need to be exact, keeping some degree of scale in the drawing is desirable.  | Use of Geogebra for checking curves after sketching is useful at this pointPast HSC Question2011 Q9c |
| Problems on maxima and minima. | Problems should involve finding of maximum and minimum values of given functions over different intervals and over their domains.The terms absolute and relative maxima and minima should be revisited to ensure that students have an understanding of how limiting the domain will affect them.Reviewing what a negative value can mean for measurement and time can help with students interpreting implicit limitations of a domain (or range)Also consider problems for which the appropriate function to be analysed is to be constructed from data given in words or on a diagram. | Geogebra can be used to graph functions with restricted domains. [A YouTube video showing graphing a restricted domain](https://youtu.be/gBigr0bpavc) (it also shows how to find maxima and minima of functions) |
| Tangents and normals to curves | Extension of 8.4 in the preliminary course using notations and techniques from this topic |  |
| The primitive function and its geometrical interpretation. | From the syllabus: “Given, the question ‘What is?’ naturally arises. Particular examples will make plausible, and a proof will show, that  is not uniquely determined but that two functions which have the same derivative can only differ by a constant.Geometrically, given the gradient function of a curve, the curve is not fixed but it is one of a ‘family’ of similar curves.”The term primitive function or anti-derivative should be used - not integration at this stage. | Geogebra can be used to illustrate this concept |