 Polynomials

| Content/applications/implications and considerations | Teaching strategies | Resources |
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| 7.1  Integer roots of polynomials with integer coefficients  The student is able to: Prove that, if a polynomial has integer coefficients and if  is an integer root, then  is a divisor of the constant term.Test a given polynomial with integer coefficients for possible integer roots.All possible integer roots of such a polynomial therefore lay among the positive and negative integer divisors of its constant term. | * Link to previous work on Polynomials in Ext 1
* The different number fields – real / complex -difference
* Students state factor and remainder theorem – and how they are used
* Construct a flowchart detailing how to factorise polynomials
* Link to factorising quadratics – what are the properties of the roots in relationship to the constant term
* Factorise and solve any polynomials over the real numbers
 | Worksheet [ [Word](https://drive.google.com/open?id=0B2rBnOj-8kBAeFJ2aUcwZjZ2eWM) | [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBATndLbmo5ME9SOWM) ]Introductory Lesson [ [Keynote](https://drive.google.com/open?id=0B2rBnOj-8kBAdzFxMnZDOGdKRk0) | [ppt](https://drive.google.com/open?id=0B2rBnOj-8kBANE54MWdmOGE2ekU) | [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBATWNnQnB4SVEzTWM) ]Undetermined Coefficients Example [ [ppt](https://drive.google.com/open?id=0B2rBnOj-8kBAb0FzYlNEcFU2Tkk) ][Youtube Playlist – all areas Polynomials](https://www.youtube.com/playlist?list=PLR5DXS1EdWhzzjYHvcKqvPUNuBJXGURGZ)[Introduction to Polynomials – Scootle resource](http://www.amsi.org.au/ESA_Senior_Years/SeniorTopic2/2_md/SeniorTopic2e.html)[Fool Proof Test for Primes - Numberphile (YouTube Video)](https://youtu.be/HvMSRWTE2mI)[Odd Equations – Numberphile (YouTube Video)](https://youtu.be/8l-La9HEUIU) |
| 7.2  Multiple Roots  The student is able to: Define a multiple root of a polynomial It may happen that  and that  is a factor of . The number  is then called a repeated or multiple root of .Write down the order (multiplicity) of a rootIf , where  is a positive integer and , then  is a root of  of order (or multiplicity) .  is called a factor of  of order . A simple root corresponds to a factor of order 1.Prove that if  where  and , then  has a root  of multiplicity . Solve simple problems involving multiple roots of a polynomial. | * Factorise and solve any polynomials over the real numbers
* Generate understanding of multiple roots using Geogebra – get Geometrical significance
* Prove multiple root Theorem
* Applications of Multiple Root Theorem – typical past HSC questions - find the multiplicity of a particular root – use multiple roots to aid in the sketching of polynomials
 | Typical Question 1986 Q3 (ii)Investigation – Geogebra Roots of Multiplicity [ [Word](https://drive.google.com/open?id=0B2rBnOj-8kBAWi12dnRzd3VZNG8) | [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBAcnpwNUxzYVprWFk) ]Roots of Multiplicity [ [Keynote](https://drive.google.com/open?id=0B2rBnOj-8kBATWNnQnB4SVEzTWM) | [ppt](https://drive.google.com/open?id=0B2rBnOj-8kBAczB3ZzdhTy1xVjg) | [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBAczB3ZzdhTy1xVjg) ] |
| 7.3 Fundamental Theorem of AlgebraThe student is able to:State the fundamental theorem of algebra The ‘fundamental theorem of algebra’ asserts that every polynomial  of degree  over the complex numbers has at least one rootDeduce that a polynomial of degree , with real or complex coefficients, has exactly  complex roots, allowing for multiplicities. Using this result, the factor theorem should now be used to prove (by induction on the degree) that a polynomial of degree  with real (or complex) coefficients has exactly  complex roots (each counted according to its multiplicity) and is expressible as a product of exactly  complex linear factors | * State the Fundamental Theorem of Algebra – no proof
* Link to the Division Transformation
* Roots Occurring in Conjugate Pairs
 | Fundamental Theorem of Algebra [ [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBAb0FzYlNEcFU2Tkk) ][Fundamental Theorem of Algebra - Numberphile (YouTube Video)](https://youtu.be/shEk8sz1oOw) |
| 7.4  Factoring Polynomials  The student is able to: Recognise that a complex polynomial of degree  can be written as a product of  complex linear factors The fact that complex roots of real polynomials occur in conjugate pairs leads directly to the factorisation of real polynomials over the real numbers as a product of real linear and real quadratic factors. In particular, a real polynomial of odd degree always has at least one real root.Recognise that a real polynomial of degree  can be written as a product of real linear and real quadratic factors Factor a real polynomial into a product of real linear and real quadratic factors Factor a polynomial into a product of complex linear factors.Students should be able to factor cubic and quartic polynomials over both the real and complex numbers. Students should be able to factor polynomials with a degree greater than 4 in cases where factors are possible to obtain by other than the remainder theorem (e.g. , )Write down a polynomial given a set of properties sufficient to define it.Solve polynomial equations over the real and complex numbers.  | * Focus more on reverse type questions – worded properties of polynomials
 | [Polynomial with Complex Roots – YouTube](https://www.youtube.com/watch?v=i3ZcRqEW4Aw) |
| 7.5  Roots and Coefficients of a Polynomial Equation.The student is able to: Write down the relationships between the roots and coefficients of polynomial equations of degrees 2, 3 and 4. Use these relationships to form a polynomial equation given its roots The simplest approach to forming an equation whose roots are related to the roots of a given equation often does not involve using the relationship between the roots and coefficients. Form an equation, whose roots are a multiple of the roots of a given equation An equation, whose roots are m times those of a polynomial equation  , is  For example, a cubic equation  may have roots . Then an equation with roots  is  Form an equation, whose roots are the reciprocals of the roots of a given equation An equation, whose roots are reciprocals of those of , is  Thus a cubic whose roots are  is  or .Form an equation, whose roots differ by a constant from the roots of a given equation An equation, whose roots are all  less than those of , is . Thus, a cubic with roots  is .Form an equation, whose roots are the squares of the roots of a given equation.An equation, whose roots are the squares of those of , is  (converted to a polynomial in ). Thus, an equation with roots  is , or . | * Link in to previous experiences with quadratics and cubics – only need to know up to degree 4 – but mention general forms as well.
* Go through standard types of root properties and generate how to get them from standard properties of roots
* Focus on working backwards – given properties of the roots finding the polynomial or polynomial equation
 | A typical question on the relationship between roots and coefficients of a polynomial was Question 7(ii), 4 Unit HSC Mathematics paper, 1982. A typical question on formation of an equation with related roots is Question 7(ii), 4 Unit HSC Mathematics paper, 1983.Roots and Coefficients Example [ [Keynote](https://drive.google.com/open?id=0B2rBnOj-8kBAT0FaM1NBY2R0blU) | [ppt](https://drive.google.com/open?id=0B2rBnOj-8kBAbllRZzJJaUFLYjQ) | [pdf](https://drive.google.com/open?id=0B2rBnOj-8kBAb0FzYlNEcFU2Tkk) ] |
| 7.6 Partial Fractions.The student is able to:Write , where , in the form , where .If , then  must be divided initially by  in order to write into a form upon which a partial fraction decomposition may be applied.Write , where  and  is a product of distinct linear factors , in the form .A variety of methods should be examined, in carrying out a decomposition of  into partial fractions, when  is a product of distinct linear factors.If , then  Coefficients may then be obtained by equating coefficients, substituting in carefully selected values of  or using the fact that .This may be derived by noting that, if  and  , then . Let  tend to . Then  and ..Write , where  and  is a product of distinct linear factors and a simple quadratic factor, in the form .Write , where  and  is a product of two different simple quadratic factors of form , in the form .Apply these partial fraction decompositions to the integration of corresponding functions. Cases when multiple factors of  existeg  are not included in this course. | * This can be left until the integration topic – but it is wise to go through now and refresh when doing integration
 | [Partial Fractions Synopsis](http://math.usask.ca/emr/pfd.html)  |