 Year 11 Mathematics Standard

Unit title: MS-S2 Relative Frequency and Probability 

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Duration: 8 hours

Rationale

Probability is of significant importance in everyday life. It is concerned with drawing conclusions related to the likelihood or chance that an event will occur. The study of probability is important in developing students’ awareness of the broad range of applications of probability concepts in everyday life, including, for example, in relation to the reliability of products, and how probability assessments are made and used in decision-making.

Topic focus

The principal focus of this subtopic is to draw conclusions related to the chance that an event will occur.

Students develop awareness of the broad range of applications of probability concepts in everyday life and their use in decision-making.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

Prior knowledge required

* Represents probabilities of simple and compound events MA4-21SP
* Calculates relative frequencies to estimate probabilities of simple and compound events MA5.1-13SP

Language considerations

arrays

complement

expected frequency

outcomes

population

relative frequency

sample

sample space

tree diagrams

Outcomes

A student:

* solves probability problems involving multistage events MS11-8
* uses appropriate technology to investigate, organise and interpret information in a range of contexts MS11-9
* justifies a response to a given problem using appropriate mathematical terminology and/or calculations MS11-10

Assessment (including formative and summative)

Some strategies for formative assessment could include:

* Reflecting on students’ responses to a class discussion
* Beginning the lesson with a few questions on content from previous lessons before progressing
* Having students write their own questions on a topic or having them write a specific number of questions with the same answer
* [3-2-1 Exit slips](http://www.theteachertoolkit.com/index.php/tool/3-2-1) - http://www.theteachertoolkit.com/index.php/tool/3-2-1
* [Chalk Talk Routine](http://www.santeesd.net/cms/lib/CA01000468/Centricity/Domain/12/VT_ChalkTalk.pdf) - http://www.santeesd.net/cms/lib/CA01000468/Centricity/Domain/12/VT\_ChalkTalk.pdf
* [Red, Yellow, Green Cups](http://www.sstr2.org/Downloads/Cups%20as%20student%20feedback.pdf). Alternatively, students could indicate beside the questions their level of understanding using the letters, R, Y or G. http://www.sstr2.org/Downloads/Cups%20as%20student%20feedback.pdf
* [Mindmaps](https://emedia.rmit.edu.au/learninglab/content/how-create-mind-map) - https://emedia.rmit.edu.au/learninglab/content/how-create-mind-map

| Content | Teaching and learning strategies and evidence of learning | Resources |
| --- | --- | --- |
| * review, understand and use the language associated with theoretical probability and relative frequency ◊ **Paperclip icon** Literacy icon
	+ construct a sample space for an experiment and use it to determine the number of outcomes (ACMEM154)
	+ review probability as a measure of the ‘likely chance of occurrence’ of an event (ACMMM052)
	+ review the probability scale: $0\leq P(A)\leq 1$ for each event $A$, with $P\left(A\right)=0$ if $A$ is an impossibility and $P\left(A\right)=1$ if $A$ is a certainty (ACMMM053)
 | Theory* Students need to ensure they understand how to calculate the basic probability and represent the probability in an appropriate numerical form.

Application* Teacher leads a class group activity with the goal of ordering a series of simple events from most likely to least likely. Events could include: It will rain tomorrow; a flipped coin shows heads; the sun will rise in the west, and so on.
* Students complete a probability scale including numerical values for probabilities and matching words to describe probabilities.
* Students to complete as a think-pair-share The think, pair, share strategy is a cooperative learning technique that encourages individual participation and is applicable across all grade levels and class sizes. Students think through questions using three distinct steps:
	+ Think: Students think independently about the question that has been posed, forming ideas of their own.
	+ Pair: Students are grouped in pairs to discuss their thoughts. This step allows students to articulate their ideas and to consider those of others.
	+ Share: Student pairs share their ideas with a larger group, such as the whole class. Often, students are more comfortable presenting ideas to a group with the support of a partner. In addition, students' ideas have become more refined through this three-step process. This could lead to class discussion around different answers.
* Students identify/construct the sample space for some common experiments in probability. For example; rolling a die/two dice, cards in a normal deck, flipping coins

Metalanguage* Students research formal definitions for key terms in probability such as: event, sample space, outcome, compound event, mutually exclusive, complementary events
 | * [A number of videos available for probability](http://splash.abc.net.au/home#!/topic/494566/chance-and-probability/video) http://splash.abc.net.au/home#!/topic/494566/chance-and-probability/video
* [Probability Scale:](https://www.google.com.au/search?q=probability+scale&safe=strict&source=lnms&tbm=isch&sa=X&ved=0ahUKEwj16-qb7bLWAhWJW5QKHXLOCeUQ_AUICigB&biw=1680&bih=919)
 |
| * determine the probabilities associated with simple games and experiments ◊ **Paperclip icon**
	+ use the following definition of probability of an event where outcomes are equally likely: $P(event)=\frac{number of favourable outcomes}{total number of outcomes}$
 | Theory* Review identifying the sample space associated with simple games. For example; rolling dice, deck of cards, simple spinners.

Application* Students learn the formal definition for theoretical probability, work through worked examples and apply the formula to calculate the probability of simple events. Emphasis should be put on the fact that probabilities may be given as fractions, decimals or percentages.
* The teacher could pose some reverse questions by giving a probability and have students come up with an event that matches. For example;  (Students may say that the event is selecting an Ace or King from a deck of cards.
* Students could play dice games to develop an understanding of the difference between experimental and theoretical probability. Students should be encouraged to decide upon a strategy after explaining the rules and before playing the game. Students should then be encouraged to reflect on their strategy and possibly refine or change their strategy before playing again. Discussion should focus on what is expected, before the game begins. Then, as the game continues, discussion should evolve to discuss firstly why the expected probability and the experimental probability are not the same (I’ve rolled 10 sixes in the last 15 rolls, does that mean I can’t get a 6 on the next roll?) and then also how many repetitions of an event might need to occur before experimental and theoretical probability begin to mirror each other.
* Students could perform a number of experiments to see if they get the expected outcome. For example; roll a dice 6 times and get 1 of each number. Students, working in pairs, could repeat the experiment set number of times. The pairs can then pool their results to allow a comparison between experimental and theoretical probability. If 10 pairs repeat the experiment 100 times each, then the total number of repetitions will be 1000. This can lead into the experimental probability later in the topic.

Metalanguage* Students should be able to identify the difference between ‘Experimental Probability’ and ‘Theoretical Probability’.
 | * [Dice Games](http://www.mathwire.com/data/dicetoss1.html) http://www.mathwire.com/data/dicetoss1.html
* PIG - After playing a number of times some discussion points may be:
	+ Is this a fair game?
	+ Is there a strategy for winning?
* Ladybug Game - Students play the game and record the winning ladybug number on a class record sheet. Discussion points can include:
	+ Is the game fair?
	+ Is one number more likely to win than another?
 |
| * + calculate the probability of the complement of an event using the relationship $P(an event does not occur)=1-P(the event does occur)=P(\overbar{the event does occur})=P(event^{c})$
 | Theory* Students need to be able to identify complementary events and use the probability of one to calculate further probabilities.

Application* Gather responses on students understanding of the word complement (for example; some that contributes extra features to something else that leads to an improvement). The teacher defines the meaning of complement in the context of probability.
* Students investigate the probability of an event and its complement to discover the relationship between the two.
	+ P(rolling a 6 on a dice) =
	+ P(NOT rolling a 6) =
	+ P(rolling a 6) + P(NOT rolling a six) =
	+ Students use complementary events to find the probability of compound events. For example; When rolling a die: P(number less than 6) = 1-P(6)

Metalanguage* Students should be encouraged to identify complementary events by thinking about the opposite event. Therefore, the opposite of ‘winning’ is ‘not winning’.
 | N/A |
| * use arrays and tree diagrams to determine the outcomes and probabilities for multistage experiments (ACMEM156) **AAM** **Paperclip icon**
	+ construct and use tree diagrams to establish the outcomes for a simple multistage event
	+ use probability tree diagrams to solve problems involving two-stage events
 | Theory* Students need to learn how to construct tree diagrams and arrays to represent multi-stage events.

Application* Students are provided with a definition and example of a multi-stage event. Students then brainstorm what types of experiments may be considered as multi-stage experiments and how many stages they have. Examples of multi stage events may be: tossing two coins, selecting one boy and one girl from a mixed class, selecting a high school student to represent the Newcastle school district by first selecting the high school, then the year group, then the student.
* As a think-pair-share, students write down the sample space for the multi-stage experiment of rolling two dice. This activity should yield a range of different strategies for how students arrange information and could lead to some interesting class discussion, such as the benefits of arranging outcomes systematically versus a disorganised group. The teacher may then choose to show that an array is the best method.
* Students solve problems involving the construction of tree diagrams and listing the outcomes (Sample Space) for the experiment, as well as calculating probabilities. For example; What is the probability of getting (at least one/exactly one) head when a coin is flipped twice?

Metalanguage* Using the Living Oxford Dictionary to view word definitions and origins to help create more understanding (keywords to be searched could include outcomes, arrays, multi-stage)
 | * [Rules of Two Up](http://www.dailytelegraph.com.au/newslocal/inner-west/come-in-spinner-learn-how-to-play-twoup-in-time-for-anzac-day/news-story/f16f8ef4df540d166f8a0b7ea7cac834) - http://www.dailytelegraph.com.au/newslocal/inner-west/come-in-spinner-learn-how-to-play-twoup-in-time-for-anzac-day/news-story/f16f8ef4df540d166f8a0b7ea7cac834
* Appendix 2: Think, Pair, Share
* [Oxford Living Dictionary](https://en.oxforddictionaries.com): https://en.oxforddictionaries.com
 |
| * solve problems involving simulations or trials of experiments in a variety of contexts **AAM** ◊ **Paperclip icon**
	+ perform simulations of experiments using technology (ACMEM150)  Information and communication technology capability icon
	+ use relative frequency as an estimate of probability (ACMEM152)
	+ recognise that an increasing number of trials produces relative frequencies that gradually become closer in value to the theoretical probability  Information and communication technology capability icon
	+ identify factors that could complicate the simulation of real-world events (ACMEM153)
 | Theory* Students should be given the opportunity to compare the theoretical probabilities and the relative frequencies from an experiment.

Application* Students complete a practical experiment rolling dice 60 times. Students then investigate and describe the effect of increasing the number of trials. Results from all students can then be pooled to observe the effect of increasing the number of trials of the experiment. Students should be able to observe that experimental and theoretical probability begin to align.
* Students use a spreadsheet to simulate the flipping of a coin/rolling a dice, and so on. Students investigate the difference between the expected probability and the experimental probability and what happens as the number of trials increases.
* Students could conduct another dice rolling game “Roll a 6”. Students count how many times it takes to roll a 6 on a single die. Record this number and repeat. Consider, what might happen as you play this game over and over again. If 1000 people play this game, what would the average number of rolls needed be? What would the median be? Which would be larger/smaller?
* Students could perform some practical experiment such as throwing balls in a bucket/shooting a basketball, recording the number of shots it takes then make some probability statements based on the results

Metalanguage* Students research the definitions of probability terms and link the terms to an example. Terms to define could include sample space, tree diagram, array.
 | * [Online simulator](http://www.shodor.org/interactivate/activities/ExpProbability/) http://www.shodor.org/interactivate/activities/ExpProbability/
* [Excel Spreadsheets](http://www.mathmammoth.com/lessons/probability_simulations.php) http://www.mathmammoth.com/lessons/probability\_simulations.php
 |
| * solve problems involving probability and/or relative frequency in a variety of contexts **AAM** **Paperclip icon**
	+ use existing known probabilities, or estimates based on relative frequencies to calculate expected frequency for a given sample or population, eg predicting, by calculation, the number of people of each blood type in a population given the percentage breakdowns
	+ calculate the expected frequency of an event occurring using $np$ where $n$ represents the number of times an experiment is repeated, and on each of those times the probability that the event occurs is $p$
 | Theory* Students should be given the opportunity to identify practical applications of probability.

Application* Students investigate blood types in countries around the world. Use this data to solve problems around the number of people with certain blood types in particular populations. Students can calculate the expected number of people in each blood group when given the percentage of people with that blood type. An excursion to the Red Cross Blood Service may be considered.
* Students can look at the cost of car insurance online. Students can alter the information, such as age, gender, value of car, and compare the different costs of insurance. Students can use the comparisons in regards to make statements about the likelihood of car crashes, theft of a car, average distance travelled. Discussion should focus on the use of experimental probability to determine the expected frequency of these events. Students could discuss loan applications and how banks calculate the degree of risk a potential customer has.

Metalanguage* Students can use metalanguage to make predictions based on probabilities.
 | * [Blood types by country](http://www.rhesusnegative.net/themission/bloodtypefrequencies/) http://www.rhesusnegative.net/themission/bloodtypefrequencies/
* [Investigate Red Cross data](http://www.donateblood.com.au/): http://www.donateblood.com.au/
* In Australia in 2012, there was the following breakdown of blood types:
	+ O positive - 40%
	+ O negative - 9%
	+ A positive - 31%
	+ A negative - 7%
	+ B positive - 8%
	+ B negative - 2%
	+ AB positive - 2%
	+ AB negative - 1%
* Students may watch the movie Moneyball and can analyse the statistics involved. Look at statistics used, what decisions are made based on the statistics and/or what other influences are based on the statistics.
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Reflection and evaluation: