 Inverse functions and inverse trigonometric functions

| Syllabus element | Teaching ideas | Teaching resources |
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| If in addition  is a differentiable function of  then (since a tangent line to  is also a tangent line to )  is a differentiable function of , and (since the relevant angles of inclination are complementary) = 1/ or = 1, which may be formally obtained from the definition of the derivative**.** | Inverse relationships are the reflection of a function in the line  .This relationship is a function if there still one unique  value for each  value.  Inverse functions are two functions that do exactly the opposite of each other. As such .  Students should be aware that the notation for an inverse function , is not the same the reciprocal notation .  The problem of defining an inverse function when the equation  has more than one solution x for a given *y* should be discussed;  In practice, most students just swap the  and  values around to find the inverse of a function. While this works algebraically, it can lead to difficulties working out domain and range.  The derivatives of a function and its inverse function are related so that . | Mira Mirrors are a great way to get students comfortable sketching the inverse function. Alternatively, using rubbings can be useful as seen in this [YouTube video of a Graph of a function and its inverse](https://www.youtube.com/watch?v=0ImB4PTrxyY).  This is shown very simply on the Geogebra page of ‘[Slope of Inverse Functions](https://www.geogebra.org/m/C3T9PyG7)’ and ‘[Derivative Rule for Inverse function](https://www.geogebra.org/m/kKfTZdEw)’. |
| Symbol confusion | It is important to realise that inverse function notation can be very confusing to students who recognise (and often want to operate) as if it is a reciprocal relationship. It should be made clear through carefully selected examples that .  Note that  is indeed !!!  Why is there this confusion with the inverse function? Because the notation  really does mean inverse not to the power of negative one. It just happens to be that  is the multiplicative inverse of . i.e. . |  |
| Exploring the domain restriction for the inverse trig functions | By reflecting the trigonometric functions in , students will quickly observe the resulting relations are not functions.  There are many domain restriction which would “work” to make then inverse *functions* - which to choose? They key idea is we are looking to include the acute angles (would not make sense if they weren’t in the domain) and then locate a monotonic region which includes the acute angles. |  |
|  | Using the above techniques and the reflective properties of inverse functions, students should be shown the inverse trigonometric relationships. Discussion should then focus on what limitations to domain and range would ensure that these could be functions.  Eventually, the following graphs and domains should be discovered:   1. for  and range 2. for  and range 3. for  and range | Geogebra is an excellent way for students to graph functions and their inverse easily.  [Geogebra tube](http://tube.geogebra.org/) has many pre-made files on inverse functions and inverse trigonometric functions including this one on [Inverse Functions of Trigonometric Functions](https://www.geogebra.org/m/DTurVvb9). |
|  | These are best shown as a graphing exercise and can be linked with previous trigonometric function knowledge:   1. (Odd function) 3. (Odd function) | Geogebra lends itself to investigating these properties as a number of graphs can be shown on the same axes and/or added together.  No. 4 can be proven algebraically by using complementary angles, . |
|  |  | [Khan Academy page on the derivate: inverse sign](https://www.khanacademy.org/math/ap-calculus-ab/ab-derivatives-advanced/ab-diff-inverse-trig/v/derivative-inverse-sine)  [YouTube video on the introduction to Inverse Trig Functions](https://www.youtube.com/watch?v=Xx8UxNprQtw&list=PL5KkMZvBpo5DVLO7J9qN2zfZ9j8VqfYhs) |
| Methods to integrate inverse trig function |  |  |