 Rates of change and exponential growth and decay

| Syllabus Element | Teaching Ideas | Teaching Resources |
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|  | Students need to be comfortable that the derivative of a function represents the rate of change of the dependent variable with respect to the independent variable. Since the world is a dynamic place, this is a very powerful concept with applications such as:* modelling economic development
* modelling business strategies
* researching the success of medical intervention
* interest rates

In the Extension course, students will deal with rates of change between more than one variable. This requires them to use the chain rule to find the required derivative. This allows us to work when we cannot ‘measure’ the variable that we are interested in. | [Investigation of rates of change using liquid pouring experiments](http://gomaths.net/2722)  |
| Planting the seed | Students will find this topic much easier conceptually if the introductory work on the derivative includes significant discussion on the connection between rates of change and the derivative, as opposed to a purely mechanical or algebraic interpretation of differentiation. |  |
| Take advantage of the scientific use of “units” | In Mathematics, we often “throw away the units”, however science teachers know they are a powerful cue as to which variables are being related to other variables. It is worth explaining to students why, for example, acceleration has units of metres per second per second.Units often help us decide how to relate derivatives and are surprisingly helpful when solving these types of problems.Units also help us determine if the correct linkage has been made between the right variables. |  |
| Confusion between constants and variables | Harder related rates of changes problems involve many parameters expressed using pronumerals. Students can have difficulty recognising which aspects of the problem are constant and which are variable. This should be explicitly highlighted in worked examples.Diagrams are essential to form the equations. | Exposition of variables versus constants: |
| Exploration of a Harder Problem | Many EGAD problems required complex situations.In these three related videos from Wootube, a challenging problem is approached three different ways (watch in order).Use this only after students are confident solving more standards Related Rates of Change problems.* Related rates: [Police Car (flawed approach)](https://youtu.be/CWajHUKstK0?list=PL5KkMZvBpo5Ahgc5EnzYPYjBqz3utcPNv)
* Related rates: [Police Car (A Better Approach)](https://youtu.be/us7KdIxC1Ao?list=PL5KkMZvBpo5Ahgc5EnzYPYjBqz3utcPNv)
* Related rates: [Police Car (Implicit)](https://youtu.be/0sGmtb4xW-M?list=PL5KkMZvBpo5Ahgc5EnzYPYjBqz3utcPNv) [Suitable for Extension 2 students]
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| , where A is an arbitrary constant. Some numerical examples should be undertaken, determining A and/or k from given initial conditions. It should be noted that whenever k < 0, the population N tends to the limit P as t → ∞, irrespective of the initial conditions. The case k > 0 should also be discussed. | Applications of EGAD include:* Epidemics
* Population growth
* Extinction of species
* Radioactive decay (see www.Maths300.com Lesson 7)
* The Greenhouse Effect (or concentrations of carbon dioxide in the atmosphere).

While many of these can be modelled using the basic equation they are often more accurately modelled using the extended equation. This is where the population growth (or decay) is not directly proportional to the population itself but to how much the population exceeds a fixed amount (graphically, this is just a shift up the *y*-axis).Memorisation of the general solution  is not required. HSC questions will always give a solution in some form, and may then ask to verify by substitution that it is a solution of the differential equation.  | [Exponential Outbreaks: The Mathematics of Epidemics](http://learning.blogs.nytimes.com/2014/11/05/exponential-outbreaks-the-mathematics-of-epidemics/?_r=0)[World population estimates](http://gomaths.net/2871)[The Australian Bureau of Statistics has real data available for analysis](http://www.abs.gov.au/websitedbs/cashome.nsf/Home/Home) |
| Simulation games | “Death of Mr Potato” – solve the problem: when was Mr Potato killed?* Raise the temperature of potatoes to a determined level (e.g. boil them in a pot until cooked, or microwave until steaming). Record this temperature. Tell students this is the “body temperature” or the “death temperature”
* Remove potatoes and allow them to partially cool.
* Bring into classroom, with thermometers.
* Students periodically record temperature of the potatoes (can be done throughout the lesson)
* Students plot curve, predict when Mr Potato died.

This experiment is analogous to a forensic “when did the victim die” problem - based on body temperature and the environment. |  |
| Exploration of the classic curves | Students should explore the difference between these equations:   Recognising the difference in the initial conditions and the limiting values/end condition. |  |
| Exploration of the “Cooling Coffee Curve” | It is helpful to develop an intuitive feel for the coffee cooling curve: |  |
| Exploration of the “Turkey in the oven” curve | Classic challenge – why is the sign in the equation negative? Answer: The turkey temperature is increasing; however as the turkey temperature gets closer to the oven temperature, the rate of the turkey temperature rise slows down.Graphing , and not just T will show this.Good news – in the HSC examination, students are usually given the general form of the starting equation, asked to validate the function satisfies the differential equation, and then asked to find values of the constants. |  |