 Motion

| Syllabus element | Teaching ideas | Teaching resources |
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| Velocity as a function of displacement  | The traditional approach is to reciprocate the derivative: |  |
|  | Students studying Extension 2 would benefit from being shown the “separation of variables” method: |  |
| Why do we have equations ? | Many of your students are also studying the HSC Physics or Engineering Studies courses - they will appreciate being shown the links to the mathematics courses.Why do we work with ? The concept of force fields (eg: gravitational, electrical fields) describe forces on particles which are dependent on *where* the particle is in the field - not *when* the particle arrived at the location, nor how it got there. Since acceleration depends on force (and mass), the acceleration of the particle in a force field is thus dependent on *position* not time. |  |
| The relationship between acceleration and   | The traditional NSW syllabus result is: definition of  the chain rule definition of  the chain ruleStudents find this challenging - most will resort to memorising the result with little understanding. This is not helpful, especially since students may be asked to demonstrate the result.Start first by finding an expression for  when given  where . Some more concrete examples may help – .Students will need to see the full derivation on several occasions and should be able to work in both directions (i.e. from Left to Right, and Right to Left). |  |
|  | Another approach is to use the expression:and then use the separation of variables technique:This is particularly suited to Extension 2 students who will use this technique in harder motion problems.Using is very helpful when given  and asked to find  The other approach requires the student to square the function, halve it, and then differentiate. |  |
| The connection with Kinetic Energy | Students with an interest in Physics may notice a link to the formula for Kinetic Energy:This is not a coincidence.The work done to move a particle through a field will be:or over a small distance in a varying field:for a fixed mass,The total work done as the particle moves through the field will beUsing the relationship : | See the [Wikipedia page on Kinetic energy](https://en.wikipedia.org/wiki/Kinetic_energy#derivation) |
| Kinematics versus Dynamics | **Background for teachers** – the motion topics in the Extension 1 course work with equations of motion as “givens”: we are told some aspect of the motion (displacement, velocity or acceleration) given function of time (2 Unit) or displacement (Extension 1) and then use mathematical relationships to find the other equations of motion as desired.This approach is called “**Kinematics**” - we have the equations, we see where they lead to. However there is no explanation for the underlying cause of the motion (ie. forces).In Extension 2, we begin the path to **Dynamics** which analyses the forces involved and then derives the resulting acceleration equation using Newton’s Laws of Motion. While an approach to motion using Dynamics is beyond the Extension 1 Mathematics course, making some reference to the dynamics of situations (forces at work) can make a significant difference in helping students understand the equations. This is particularly the case for Simple Harmonic Motion (see below). |  |
| General teaching commentary | Another topic that is typically treated very briefly in most text books but warrants a full week of teaching time. Problems range from simpler entry level questions where students develop equations of motion to much harder problems which may involve complex algebraic manipulation.For the harder questions, a key element is often being able to use properties of the resulting quadratic equations, such as the sum and product of the roots and properties of the discriminant. A clear sign the question is going to be harder is when the equations of motion are provided up front. |  |
| Key idea: What is a projectile? | A projectile is an object that is launched or dropped and has no forces acting on it other than gravity (we are ignoring air resistance [in Extension 1]). There are no other forces producing the motion such as a jet engine or a propeller. | If you would like to see the effect of air resistance, see [Brian Cox’s video on projectiles in vacuums](https://www.youtube.com/watch?v=E43-CfukEgs). |
| Key idea: Resolving the  and  dimensions (vectors) | We can resolve all the 2 dimensional equations of motion (displacement, velocity, acceleration) into two completely independent (orthogonal) sets of equations, one for the  direction, one for the  direction.In effect, the motion in the  and  direction are completely independent of each other. We develop parametric equations where time is the parameter. Later we can find the locus of these independent sets of equations, to find the physical equation of the path. |  |
| Key idea – find the equations of motion | Everything hinges on two key observations:* There is no acceleration in the  direction. In other words, nothing changes the  velocity - the  component just keeps on moving at a uniform rate.
* There is a constant acceleration downwards in the  direction (value , usually rounded to 10).
 |  |
| Exploring Projectile Motion | Students should explore what happens when:* The launch angle changes for the same initial speed
* The initial speed is altered for the same launch angle
* Attempting to hit a target in the air for a fixed initial speed - in some conditions 2 solutions are possible.
 | [Explore at PhET: “Projectile Motion” Launch pianos and cars from a canon!](http://phet.colorado.edu/en/simulation/projectile-motion) |
| Misconceptions: Shouldn’t a heavier object fall faster? | Why does everything fall at the same rate under gravity, when clearly heavier objects are, well, heavier. Doesn’t gravity exert a greater force on them, so shouldn’t they accelerate faster?It is true there is a greater force acting on the heavier object. But the heavier object has more *mass*, which means it takes more force to accelerate it. | [YouTube – Veritasium: Misconceptions About Falling Objects](https://www.youtube.com/watch?v=aRhkQTQxm4w) |
| Misconceptions : The Monkey and the Hunter | Students have difficulty accepting the  velocity component has no effect on the  velocity or position. It makes no difference what the  velocity it, the projectile falls at the same rate. It doesn’t matter if the object is a bullet or snail. They will both land at the same time. The difference is the bullet will be a lot further across (in the  direction) than the snail when they land on the ground.A classic thought experiment is “The Monkey and The Hunter” | [Physics Central:The Monkey and the Hunter](http://www.physicscentral.com/explore/action/monkey-hunter.cfm)[YouTube:MIT Physics Demo -- Monkey and a Gun](https://www.youtube.com/watch?v=cxvsHNRXLjw) (no monkeys were harmed in this video) |
| Deciphering Projectile Motion Questions | Students should develop the ability to recognise and translate the following key phrases:“Find the maximum height of the projectile” means find where the  velocity is zero“Find total time of the flight” means the value of  which gives the final  value. For a level ground, this means, find when .“Find the range” means find the total time of the flight (see previous item), then find the  position for that time.“Find the speed and the angle of inclination when the projectile lands” means find the  and  velocity components at this time and then find the magnitude and angle of the resulting velocity vector or using the equation of the path, find the size and angle of the gradient at the end point. |  |
| Using Geogebra to model solutions | To view the path given the time equations, use the Curve[] function.For example, set up parameters  and  (can be done with sliders), then: |  |
| The Equation of the Path | After derivation of the equation of the path, it is important to show the equation is a quadratic in  as much as it a quadratic in . |  |
| Student Difficulties | Students who have not done enough practice questions will forget to use the identity  when trying to solve for equations of the path. |  |
| General teaching commentary | Students find this topic challenging and will need time to digest it. Many of the more complex topics are brought together here: trigonometric functions with radian measure, the auxiliary angle transform, motion in terms of displacement and the general solution of trig equations. Given that most students are still consolidating these skills, it is easy to see how Simple Harmonic Motion can be challenging.Each idea in the topic can be seen from several perspectives and a key challenge is for students to select the most efficient way to approach different problems - choosing a time or a displacement approach, choosing how to tackle the trigonometry. Most questions have implied information that is not explicitly stated. A classic example is the HSC Mathematics Extension 1 2006 Question 4(b) - ostensibly a very simple question, but according to the markers was poorly done by most students.Most text books compress a lot of content into one or two exercises for a topic which realistically needs a whole week. Teachers new to this topic will find it takes several years of teaching it to gain deep understanding and proficiency. Once the teacher has developed this synthesis, it is easy to forget just how complex this process was - our students however have only a few weeks to digest this topic as they rush to complete the HSC course. Repeated, visible exposition of your thought processes as you interpret problems is recommended.A fundamental skill students need to develop is to translate and extract the key information from a problem stated in words, combine this with their knowledge of SHM to then write the starting mathematical equations. This is typically the hardest part of most problems. In most cases, the first step to solving an SHM problem is to draw a diagram of the motion on the x-axis and mark the significant information given on the axis. More complicated problems may also require a rough graph or a unit circle diagram to assist in the choice of quadrants when solving trigonometric equations. | HSC Mathematics Extension 1 2006 Q4(b)A particle is undergoing simple harmonic motion on the x-axis about the origin. It is initially at its extreme position. The amplitude of the motion is 18 and the particle returns to its initial position every 5 seconds.* Write down the equation for the position of the particle at  seconds.
* How long does the particle to move from a rest position to the point half way between the rest position and the equilibrium position?

**Notes from the marking Centre** – this part was very poorly done. Many candidates knew that part (i) required some version of  or , but failed to find the correct  or . A number of candidates did not know where to start. Even though ‘follow-through’ marks were awarded for part (ii) answers consistent with incorrect answers to part (i), many candidates used 0 or 18 as  rather than 9, or they differentiated first, showing an inability to translate the given words into a mathematical form. A common error was obtaining  by assuming the velocity was constant, or  because candidates found  From Q4(b), New South Wales Board of Studies, HSC Mathematics Extension 1 exam, 2006.  |
| Exploring SHM with the PHET Moving Man simulator | Use the PHET “Moving Man” simulator.Select the “Special Features/Expression Evaluator” to set This can be done as a student exploration and/or an interactive whole class activity. Note the tool has two modes: “Record mode” and “Replay Mode”. There is a cursor in Replay Mode which is very helpful to highlight and compare position, velocity and acceleration at a specific time.All the key concepts in SHM can be discovered using this simulation.A sequence of questions to ask:* How do we change the motion equation so the man hits the house and the tree? (different amplitude)
* How can we make the man oscillate faster? Slower?
* What happens if we change the equation to ?
* How could we start the journey somewhere else on the path?
* Where is the velocity zero? Where is it maximum?
* Where is the acceleration zero? Where is it maximum?
* What parameters affect the velocity? (Change the amplitude only, observe then change the frequency only, observe).
 | PhET the Moving Man simulatorThis Java application can be downloaded and run locally as a demonstration, or used in a student-led activity.The application has two modes: Record and Replay. The Replay mode is very useful for carefully analysing a motion sequence. Use the Special Feature/Expression Evaluator to program specific equations of motion. Note the expression evaluation is quite primitive and space sensitive. |
| Five forms of the time equation | Students need to be aware they have to make choices about which is the most appropriate form to use for a simple harmonic problem. Any of the forms may be used, but some are much easier to use than the others! – use when the motion starts at  – use when the motion starts at - use when the motion starts anywhere else. is extremely useful when we have the values of  and  when.Students will need to be reminded of the Auxiliary Angle Transform: etc. |   |
| Why is it called Simple Harmonic Motion? | The oscillations are simple – only one frequency, no dampening and in one dimension only.The word harmonicis a reference to music – the harmonic series of frequencies produced by oscillating strings.Students may ask: If most oscillation isn’t simple harmonic, why do we study it? More advanced techniques (Fourier Analysis) allows us to take more complex wave patterns and decompose them into discrete frequencies. So the SHM analysis is the first part of the development of more complex analysis.Potential engineering students will be interested to know that they will be designing oscillating electrical circuits, where the frequency depends on their choices of resistance, capacitance and inductance values.Mechanical oscillations and resulting catastrophic effects will be of interest to student (Tocama Narrows Bridge collapse is the most infamous). | Some examples of non-harmonic motion which exhibit the complexities of oscillations (and are great fun to watch):Cymbal being struck - high resolution slow motion video“[Vibration. See the unseen: Cymbal at 1,000 frames per second](https://www.youtube.com/watch?v=kpoanOlb3-w).” [Slow motion jello drop](https://www.youtube.com/watch?v=FH7aSxn-nEQ)Students interested in the production and hearing of sound will find this Vi Hart video interesting (13 minutes):[What is up with Noises? (The Science and Mathematics of Sound, Frequency, and Pitch)](https://www.youtube.com/watch?v=i_0DXxNeaQ0) Just for pure fun:[CYMATICS: Science Vs. Music - Nigel Stanford](https://www.youtube.com/watch?v=Q3oItpVa9fs) [Veritasium: Pyro Board: 2D Rubens' Tube!](https://www.youtube.com/watch?v=2awbKQ2DLRE) |
| A Dictionary of Terms | Centre of motion/Centre of Equilibrium |  |
|  | Amplitude - the range (difference) from the centre of motion to one extreme. The Amplitude is the  in the.  |  |
|  | Period - the time to complete one full cycle.  |  |
|  | Frequency - the number of cycles per second. Most mathematics questions use period rather than frequency. |  |
|  | Phase - the value of the argument to the trig function. Draw an analogy to the phases of the moon. |  |
|  | Initial Phase - the value of the argument to the trig function at . (Did we start at no-moon, full-moon, or half-moon?) |  |
|  | Phase Shift - the translation observed in the shifted graph of the trig function. Note that for  the phase shift is. (What makes the sine term zero). |  |
| Phases of the motion | Draw lots of diagrams to explore the phases of the motion. It is helpful to divide the motion into four parts, relating them to the quadrants in the unit circle and to the graphs of the equations.Observe that at any one specific location (apart from the extremes), there are two velocities possible - corresponding to the same speed, but in opposite directions. This will help students understand the issues of sign when the  equation is derived.Repeat this diagram for the initial condition  and . This will help students with “Low Tide/High Tide” type questions where they will be required to interpret which quadrants to look at for solutions. |  |
| The SHM Relationship  |  |  |
| Velocity and Acceleration in SHM at the centre and at the extremes | It is essential students gain a solid understanding that:* At the centre of motion, the speed is maximum, the acceleration is zero
* At the extremes of motion, the speed is zero, the magnitude of the acceleration is maximum.

There are many ways to arrive at this understanding and all of the following should be explored during the teaching sequence:From a mathematical viewpoint:* Looking at the graph of  it is clear the gradient magnitude is greatest at  - hence the speed is maximum. The gradient is clearly zero at the turning points - hence speed is zero there.
* The repeated differentiation of the  function produces these results.
* Once students have derived  we can immediately see the range of values for  and when the max and min occur.
* The equation  clearly shows where the acceleration is zero, and when it has greatest magnitude

From a physics viewpoint - imagining a mass on a spring:* When the spring is neither compressed nor extended , the spring is not applying any forces. No force means no acceleration.
* When the spring is maximally extended or compresses, the spring is apply maximum force to pull the mass back to the centre. Force is maximum, so acceleration is maximum
* Velocity has to be zero when the mass is at the extreme and about to change direction.
* As the mass moves back to the centre, it is always accelerating - so the speed is increasing. As soon as the mass passes the centre, a reverse force is applied, slowing down the mass. So the maximum speed must be at the centre.

Students should draw sketches of the motion on a number line and annotate the key positions with their corresponding velocity and acceleration. |  |
| Shifting the centre of equilibrium | The equations become: - use when the motion starts at - use when the motion starts at From a physics point of view: *nothing* changes when we change the coordinate systems!From the mathematical point of view: Since we are differentiating, nothing changes for the velocity and acceleration equations. Our graph of  has just moved up (or down).Note: The amplitude has *not* changed. |  |
| The $v^{2}$ equation | Students should be able to derive the result .It is probably not useful to memorise this result as it only applies when the centre of motion is at , but they should recognise it.The locations of minimum and maximum speed should be observed in this equation. |  |