Teaching measurement

Early Stage 1 and Stage 1

PROFESSIONAL SUPPORT AND CURRICULUM DIRECTORATE

NEW SOUTH WALES DEPARTMENT OF EDUCATION AND TRAINING

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Acknowledgements
Dr Lynne Outhred, Macquarie University, for her contribution to the development of this document.

Board of Studies, NSW, for permission to include the outcomes from the *Mathematics K–6 Syllabus*, 2002, Board of Studies, NSW.

Graphic design: Aston Hunt Design Services

Teaching measurement: Early Stage 1 and Stage 1

ISBN 0 7313 8258 7

SCIS 1134548

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Foreword

When I was younger, I remember watching my mother carrying out a strange ritual in a shop when purchasing curtain material. She would repeatedly draw the material across in front of her face from the tip of her nose to the end of her outstretched arm. I was later to learn that this ritual was known as measuring.

In order to measure, we need to know what it is that we are to measure. That is, what attribute will be measured. For my mother measuring curtain material, the attribute was the length of material. Having determined the attribute we wish to measure we need to select a regular unit. My mother's choice of unit was the yard, a formal unit that she estimated as the distance from the tip of her nose to the end of her outstretched arm. I had watched, fascinated, as she repeatedly used the unit.

Many mathematical ideas that we take for granted turn out to be significant insights for students. The idea of repeatedly using a unit when measuring is one of these. *Make*, *mark* and *move* are central to the repeated application of the unit of measurement.

Measurement is at the heart of so many things that we do. Telling time, estimating distances, cooking and making various comparisons all involve measurement. The history of measurement suggests that the art of measuring is closely linked with the advancement of knowledge. Indeed, the question, "Have we improved?" cannot be answered without the use of measurement.

A resource such as *Teaching measurement: Early Stage 1 and Stage 1* relies upon the contributions of many people. *Teaching measurement: Early Stage 1 and Stage 1*, which has developed from the *Count Me Into Measurement* program, is a blend of practical teaching ideas and research-based knowledge. The work of Dr Lynne Outhred of Macquarie University and Diane McPhail in developing *Teaching measurement: Early Stage 1 and Stage 1*, illustrates the value of combining practical research and research-based practice in mathematics teaching for our schools.

I commend *Teaching measurement: Early Stage 1 and Stage 1* to you as a resource to assist with the implementation of the new *Mathematics K–6 Syllabus*.

Robert Randall
Director
Professional Support and Curriculum Directorate

Contents

About this book

Teaching measurement: Early Stage 1 and Stage 1 is a resource designed to help teachers to plan practical, meaningful programs in the mathematics strand of measurement. Important components of this resource are its emphasis on knowledge of units and their structure (for spatially-organised units), practical activities, recording, estimation and questioning.

The material in this book is based on a conceptual framework that reinforces the similarity of the measurement processes across the different quantities, especially those quantities where the units are spatially organised (length, area and volume).

Although measurement of mass is included in this book, units of mass are not organised spatially so the framework for mass is set out separately, even though many of the processes are similar to those for the other quantities.

The measurement framework is organised into levels of increasing difficulty, each focusing on a different aspect of learning about measurement. The activities that accompany each level of the framework are designed to develop students' knowledge of the ideas of measurement, as well as the procedures and skills involved in measuring.

Teaching measurement: Early Stage 1 and Stage 1 is organised into an introductory section, followed by four main sections: Length, Area, Volume and Mass.

The introductory section provides:

- Information about teaching and learning measurement

  Fundamental measurement processes (knowledge of attributes, conservation, identification of units and unit iteration) and important aspects of teaching measurement (estimation, recording and questioning) are described.
- A detailed overview of the measurement framework

  The organisation of the Early Stage 1 and Stage 1 framework into three levels, which are similar for the measurement of each quantity, is shown. Each level is divided into two subsections and these describe the development of each attribute.

The main sections related to length, area, volume and mass each contain:

- An information section

  The knowledge and strategies to look for when students engage in the measuring activities related to each attribute.
- Lesson ideas

  Classroom activities that are designed to develop the knowledge and strategies for each level of the measurement framework. A variety of activities are included to provide opportunities for consolidation and assessment. Each activity is referenced to the measurement and working mathematically outcomes of the *Mathematics K–6 Syllabus*.
- Lesson plans

One complete lesson plan for each subsection and attribute is provided as a model. The lesson plan includes examples of the types of questions that might be asked to assess students' knowledge of key concepts.

Teaching and learning about measurement

Measurement enables continuous quantities, those which are not separately countable, to be compared and ordered. A fundamental difference between measuring and counting a discrete quantity is that in measurement the units are not visible unless "concrete" units are used or the units are constructed or drawn. The items in discrete quantities such as a box of apples or a group of children can be individually counted. To measure a continuous quantity, such as the length of a desk, the length has to be partitioned into units that can be counted by either repeating the unit along the length, or subdividing the length into units of a given size.

This book focuses on length, area, volume and capacity and mass. Measurement of some of these quantities is spatially organised. In length, area and volume the units fit together in a spatial pattern, whereas in measurement of capacity and mass the spatial arrangement of the units does not matter. Learning how spatially organised units fit together, and how they may be counted systematically, is basic to understanding the measurement of length, area and volume. To obtain a precise measurement, units must be aligned or packed so that there are no gaps or overlaps. Although capacity (fluid measure) is a measure of volume, finding the capacity of a container by filling it with liquid or material such as rice or sand is different from packing a container with cubic units, which must be organised spatially.

When informal or non-standard units such as hand spans, paperclips or popsticks are used to measure a length, the units have to be either aligned along the length, or one unit has to be repeated and the endpoint of each length marked in some way. However, when formal units are used to measure length, the measurement can usually be read from a scale on a ruler or tape, which shows units of a particular size. If students are not shown the relationship between the informal and formal measurement procedures, they may not understand the principle underlying the use of a ruler. Similarly, measuring areas and volumes with informal units assists students to understand the calculation formulae when these are taught, providing the principles underlying the informal and formal processes are understood.

Fundamental measurement ideas

There are a number of fundamental ideas that students need to learn to apply to all the measurement concepts they will encounter in the primary school syllabus. These ideas include an understanding of attributes and conservation, and knowledge of units and unit iteration.

Identification of the attribute being measured

The first step in teaching measurement is to compare quantities by direct comparison. For example, two students might stand back-to-back and decide who is taller. Comparing quantities directly helps students to identify what attribute is being measured.

Students learn what a length is by comparing it with other lengths and they develop the concept and associated language together. For example, "This stick is long but this one is short. This one is shorter." As students compare quantities directly, and order them, they learn to identify each attribute and to see how they differ. However, what is being measured is not always clear—students may confuse length and area because they are not sure which part of an object or surface is being measured. Similarly, students may think that the larger the volume of an object, the more mass it will have because they do not know the difference between mass and volume. Foam packaging is a good example to show that a large volume of material can have a small mass.

Knowledge of units is fundamental to the process of measuring

Once students are able to identify the attribute being measured, and can directly compare and order quantities, the next step is to learn to use measurement units. Units enable us to measure and compare quantities that are physically separated in time or space and to give numerical values to quantities. Once a number is associated with a quantity, that quantity can be compared with other quantities and ordered more easily than by using direct comparison. Theoretically, the quantity has to be subdivided into identical parts (units) and the number of units used gives a measurement of quantity. However, when students begin to measure they do not subdivide the length, instead they align units until they have made the required length. This process is conceptually quite different from subdivision.

A fundamental principle of measurement is that quantities can only be compared if the units used to measure each quantity are identical. Students can be assisted to develop this principle through discussion of results when different-sized units are used. Another important idea about units is that use of smaller units gives increased precision.

Any measurement is always approximate, because continuous quantities can theoretically be partitioned into smaller and smaller units, such as from metres to centimetres to millimetres and even finer units. The accuracy of a measurement can be affected by the precision of the measuring instrument, the experience of the person who is measuring as well as other factors related to the quantity being measured.

The principle of conservation is fundamental to understanding measurement

As students begin to measure with units they gradually learn an important principle of measurement, that the quantity is unchanged if it is rearranged (conservation). Students who do not understand the conservation principle may think that string is not the same length when it is curled up as when it is stretched out, or that a cup of water poured into a tall, thin glass is more than when it is poured into a short, wide glass. Nor will they realise that if a square is cut into two pieces to make a long rectangle, then the two shapes have the same area.

While an understanding of conservation is fundamental to the measuring process, this concept seems to develop from activities involving measuring, rather than being a prerequisite to measurement. For volume, conservation may not be established until later because of the complexity of volume measurement. Some students will need more experience than others in measuring quantities before they are convinced that a length, area, volume or mass measurement remains the same after the quantity is rearranged. If students measure inaccurately or use different units, their measurements will differ, making it even more difficult to grasp the principle of conservation.

Knowledge of unit iteration is fundamental to the process of measuring spatially organised quantities

A key measurement understanding for spatially organised quantities, such as length, area and volume, is an awareness of the structure or pattern of the units. Identical units are repeated or iterated so that they do not overlap and there are no gaps between them. Units may be aligned along a length, constructed in an array to measure the area of a rectangle, or packed into a container to find its volume.

Knowledge of the spatial structure of the unit iteration may help students to link concrete, pictorial and symbolic representations of measurement concepts. Once students have realised that the process of exhaustively filling a space with units is a form of partitioning that space, they may be able to re-conceptualise the space in different ways. When students think of measurement as a process of subdivision, they are no longer dependent on concrete representations of the units. They can visualise and work with abstract quantities, enabling them to manipulate fractional units and use the power of the formulae.

Rectangular shapes or containers are used when covering shapes or packing containers with units. It is important that students develop an understanding of the structure of unit covering in area and unit packing in volume. Rectangular shapes or containers assist students to see the structural relationships and usually avoid the complexity of fractional units. However, measurement provides a rich context in which to develop fractional ideas.

Important components of teaching measurement

The activities in this book are based on familiar experiences and contexts. They provide the basis for understanding the measurement process so that mathematical generalisations can be made. An important aspect of such activities is reflection, so in many of the activities students are encouraged to estimate, then measure, and finally to record their results. The questions that teachers ask to encourage students to describe, explain and justify their results are crucial.

Estimation

Estimation is seen as an essential part of measurement, because it assists students to develop a sense of the size and structure of the units. The process of estimating may also assist students to understand measurement variability and that measurement is a process of increasing precision. When students measure using informal units, such as their height or hand span, they are building estimation skills. Students need to share estimation strategies and to discuss ways to obtain more accurate estimates. These include:

- using a referent or known quantity as a comparison measure, e.g. "the dog is shorter than me" or "the seat is about twice as long as me"
- chunking or breaking a quantity into more manageable parts by estimating a distance as several shorter sections (the distance from the floor to the top of the door is about … and the distance from the top of the door to the ceiling is about…)
- "unitising" or subdividing a quantity into smaller equal parts, such as estimating the height of a ten-storey building as ten times the estimate for one-storey.

Sharing strategies for making estimates encourages students to think of an estimate as an informed, but informal, form of measurement rather than a "guess". If students predict before they measure, they will learn to judge the relative size of the quantity and the units. Estimation of two- and three-dimensional quantities (area and volume) is more difficult than estimating length.

Recording

As well as encouraging reflection, the recording process is essential as a form of assessment and as an incentive for students to develop the precise language they need to discuss measurement concepts. Common text types (procedures, recounts and explanations) can be consolidated and extended by asking students to write about what they did in measurement.

In addition to writing about their findings, students may be asked to draw their method of measuring. Drawing is seen as a bridge to link the practical activities to diagrams and plans. Drawing the array structure for the tessellation of area units appears to assist students to perceive the rows (and columns) as composite units and it is this perception that enables them to connect side length and area. If students have drawn and talked about the structure of an array, then the structure of three-dimensional packing may be grasped more easily.

Questioning

A crucial part of a teacher's role is to develop students' ability to think about mathematics. To develop thinking processes teachers need to ask higher-order questions that require students to interpret, apply, analyse and evaluate information, rather than questions that simply require students to recall facts. There are a number of strategies that teachers might use.

- Before giving a lesson, decide what the students are to learn and the key questions that will indicate if they have learnt the concept, skill or strategy that is to be taught.
- Ask probing questions that help students to clarify their responses, to see the relevance to other ideas, to be more accurate or to explain or justify why it is so.
- Encourage students to ask questions of each other so that they begin to develop independence and maturity of thought. Before students ask questions they need to consider what they may not understand or what they do not agree with in an explanation.

The measurement framework

The key levels in the measurement framework are organised into a progression that is similar for measurement of each quantity. Each level is divided into two subsections and these provide the organising framework for the development of each attribute.

The conceptual levels are:

- **Identification of the attribute to be measured**

  The student recognises the quantity to be measured and makes direct comparisons of size.

- **Informal measurement**

  Students choose and measure with informal units (given as many as they need) to compare quantities.

- **Structure of repeated units**

  Students are given only ONE unit with which to measure. Students construct the unit iteration and describe the spatial structure of length, area and volume.

The three levels in the measurement framework provide a conceptual sequence for teaching length, area, volume and capacity and mass. However, students are not expected to be at the same level in each strand. Measurement of area and volume would be expected to develop later than measurement of length, as length is the basis for measurement of area and volume.

The measurement framework

Level	Length	Area	Volume and Capacity	Mass
	Identification of the attribute			
1.1	Make direct comparisons of length	Make direct comparisons of area	Make direct comparisons of volume or capacity	1.1 Make direct comparisons of mass
1.2	Order two or more lengths by direct comparison	Order two or more areas by direct comparison	Order two or more quantities by direct comparison	1.2 Compare and order objects by hefting
				1.3 Compare masses using an equal arm balance
	Informal measurement			
2.1	Choose and use appropriate units for measuring length	Choose and use appropriate units for measuring area.	Choose and use appropriate units for measuring volume and capacity	Choose appropriate units and use them to measure a mass
2.2	Compare and order lengths by using identical units for each length	Compare and order areas by covering each area with identical units	Compare and order volumes and capacities by filling or packing with identical units	Compare and order masses using identical units for each mass
	Structure of repeated units			Relationship between units
3.1	Use one unit to work out how many will be needed altogether when making indirect comparisons	Use one unit to work out how many will be needed altogether when making indirect comparisons	Use one unit or composite unit to work out how many will be needed altogether when making indirect comparisons	
3.2	Explain the relationship between unit size and the number of units used to measure length	Explain the relationship between unit size and the number of units used to measure area	Explain the relationship between unit size and the number of units required to fill or pack a container	Explain the relationship between unit size and the number of units required to balance a mass

Level 1 Identification of the attribute

At Level 1, non-numerical comparisons are made to focus attention on the attribute being measured. Knowing what defines the attribute being measured is complex for young students. For example, students may think of length as an attribute of an object, but not as the distance between two points. They may focus on one dimension (length) when comparing areas or volumes or they may confuse quantities, such as length and area, or volume and mass.

At Level 1 when quantities are introduced they are compared directly to enable students to clarify the attribute, to discuss relative size and to practise specific terminology. The words "bigger" and "smaller" are not precise terms and can contribute to students' confusion about quantities. "Bigger" could mean an object was longer than another, one of its surfaces had more area, or that it had a greater volume or a larger mass than another object. In addition, students measure lengths and areas as parts of three-dimensional objects, so the students have to be clear about which part they are measuring.

The concept that lengths can only be compared if the ends are aligned (establishing a "baseline") is also important at this level, as is superimposition of areas to compare size.

Level 2 Informal units

At Level 2, students choose as many informal units as they need and use them to measure and compare quantities. It is important to establish the procedure of aligning, covering or packing units of length, area and volume, and to continue to develop language and recording skills.

Informal units are used for two reasons. The first is to emphasise that different units can be used to measure the same quantity but that identical units must be used when quantities are compared. The second reason is a practical one: standard units, such as centimetres and metres, are difficult for young students to manipulate.

When measuring with informal units, the attribute of the unit that is being used to measure should be emphasised. If three-dimensional shapes, such as cubes, are used to measure a length, students need to know which part of the cube is being used as the linear unit because the cube itself is a measure of volume and its faces are units of area. If cubes are used, the teacher should highlight an edge as the unit of length.

Level 3 Structure of repeated units

At this level students are given only ONE unit with which to measure so they have to construct the pattern (or structure) of the units by drawing or visualising. Tracing units is not sufficient for Level 3 understanding because students can trace units without having knowledge of the spatial structure of the units. This level does not seem to have been included previously in measurement teaching programs.

As students construct the unit iteration they learn to describe the spatial structure of length, area and volume. For example, they may construct composite units of area (rows and columns) or volume (layers). Students are also introduced to standard units of length (decimetre) and area (a 10 cm x 10 cm tile). Because of the complexity of volume concepts, the emphasis is on developing the structure of rows, columns and layers. However, a 100 millilitre scoop is introduced to lead into formal units of capacity. In this book, capacity is used for liquid measure (mL and L) while volume is used for interior space and the space an object takes up (exterior volume and displacement).

In addition to the structure of how units fill space, the relationship between unit size and the number of units required is developed. For example, twice as many new length units (5 cm long) will be required if the original length unit (of 10 cm) is halved. Such relationships are more complex for area and volume than for length, and will need to be built up using practical experiences with concrete materials.

Before you begin

You might like to start by looking at the knowledge and strategies described in the framework. Make an estimate of what your students already know and try a lesson at that level. The results will indicate if the level is too easy or too difficult or your intuition was correct.

Check that students understand the language of measurement used in the lesson, as it is crucial to learning any concept. Language prompts, such as "how far", "how many", and "how much" might be used. Students need to know the language of comparison and they should always be asked to give the units of measurement, such as four paper clips long, six of these squares.

When appropriate, ask students to demonstrate how they will measure. Sharing approaches is valuable as strengths and weaknesses of different methods can be discussed. You might demonstrate an inaccurate or impractical method and ask students how they could improve it.

Programming

It is suggested that you plan and present a block of four or five lessons based on one substrand, before moving on to a second or third substrand. This programming strategy will assist students to consolidate the essential knowledge of attributes, correct terminology and measurement techniques for each quantity. You should ensure that students are familiar with the concepts of measuring length before the activities for area and volume are commenced. When using the measurement framework as a guide to sequence lessons, remember that students are not expected to be at the same level in each substrand.

Assessing students' understanding of measurement

The framework provides several different but related activities at each Level. This enables assessment of similar concepts as students complete different activities. Students in one group could be individually questioned on one day as they work on one activity; with a different group of students completing one of the related activities becoming the focus for observation in another lesson. Encourage students to record their findings. These make excellent work samples for explaining and sharing methods, reporting back, student portfolios and displays, or they can be taken home to share with parents.

Glossary

Area	Part of a two-dimensional surface (plane or curved) enclosed within a specified boundary or geometric figure. The measure or extent of such a surface, or part of it (measured in square units)
Array	A set of elements arranged in a pattern of rows and columns
Attributes	Properties or characteristics
Capacity	A term used for a measure of internal volume (often used for liquid measure and given in litres or millilitres)
Conservation	The principle that a quantity (length, area, volume, mass) remains constant during rearrangement or reorganisation
Discrete	Individual parts or items
Displacement	The volume or the mass of fluid displaced by a floating or submerged object is equal to the volume or the mass of the submerged body
Estimate	An approximate judgement or calculation of an amount of something
Iteration	The act of doing something repeatedly, repeated application of a procedure
Layer	A thickness of material spread on or placed on a surface
Length	A measure of a line segment that is unaffected by changing the orientation of the line
Row or column	Numbers or objects arranged in a straight line (the convention for an array or grid is that rows are horizontal and columns are vertical)
Tessellation	A covering of a surface with identical shapes (a paving). The shapes fit together without gaps or overlap and can be extended infinitely in any direction

Teaching measurement: Length

Length

Length is usually one of the first measurement concepts students encounter. An understanding of length is crucial, as it is the foundation for building concepts of area and volume. Sometimes students can measure lengths without really understanding what a length is. Length can be a property of an object or shape, as in the lengths of the sides of a triangle or the edges of a desk. Length can be a height of a tree or a student, or it can be a distance, such as from the desk to the door, or from the school to the station.

The three-dimensional nature of the object being measured may obscure the linear nature of length. If the height of a tree or a person is being measured, what is really being measured is an imaginary line, which is perpendicular to the ground or floor, and joins a point on the ground or floor to a point on the top of the tree or the top of the person's head. Using a string pulled taut to measure heights or distance may help students imagine such a line. In most real-life contexts, the line that is being measured has to be imagined and the person measuring has to decide where and how length will be measured.

Teachers can model how to measure length so that students understand the need to take care when they align units, particularly when using one unit. Discussing which methods of measuring are more precise than others will emphasise the importance of keeping the size of the unit the same. Common errors made by students include putting finger spaces between the units or moving the unit without marking the end of each move carefully.

Some students learn a procedure to measure lengths by aligning one end of the ruler with the object and reading the number that corresponds to the other end of the object. In this way, students can use a ruler without knowing how its scale is constructed. Students may not be sure whether to measure from 0 or 1 on the ruler. Frequently students think that the marks, instead of the distance between the marks, are the units of measure. The experience of making a ruler by choosing, marking and numbering the informal units may assist students to understand how a ruler works.

An understanding of geometrical properties can be important in the measurement of length. When students measure a table or a desk, they usually measure along one edge. Some students may not realise that the length of a rectangular desk will be the same if it is measured along any imaginary line parallel to the edge.

Lengths can be added together and when measuring a length that is not in a straight line, such as the perimeter of a shape, each part can be measured separately and the lengths added together. Longer distances may be measured with a trundle wheel but students may need to be convinced that one rotation of the wheel is the same length as a metre ruler.

Level descriptions for length

Level 1	Knowledge and strategies
L1.1 Identification of the attribute Make direct comparisons of length	1. use length vocabulary, e.g. *long*, *high*, *tall*; *short* or *low*; *the same as* 2. put two lengths side by side to compare their lengths 3. straighten a curved or bent length to check if two lengths are the same
L1.2 Identification of the attribute Order two or more lengths by direct comparison	1. use comparative language, e.g. *longer*, *higher*, *taller than*; *shorter* or *lower than*; *the same as*; *shortest*, *longest* 2. ensure that ends are aligned for comparison by establishing a baseline 3. compare lengths systematically and explain why a length fits into a particular ordering
Level 2	**Knowledge and strategies**
L2.1 Informal measurement Choose and use appropriate units for measuring length	1. align identical units end-to-end along a given line without overlapping or leaving gaps 2. state or record that the length is the number and type of units used 3. use approximate language to explain parts of units, e.g. "about half a unit" 4. measure a circumference using string or paper strips, without overlapping ends
L2.2 Informal measurement Compare and order lengths by using identical units for each length	1. choose identical units to measure lengths 2. know that the longer line has more units 3. estimate the number of units and explain the estimation strategy 4. know that the length is conserved if rearranged
Level 3	**Knowledge and strategies**
L3.1 Structure of repeated units Use one unit to work out how many will be needed altogether when making indirect comparisons	1. measure precisely by repeating one unit 2. know that lengths (not marks or spaces) are counted 3. use a 10 cm strip as a unit to measure length
L3.2 Structure of repeated units Explain the relationship between unit size and number of units used to measure length	1. explain the relationship between unit size and number of units 2. use the metre as a unit to measure lengths 3. know that measurement techniques must be consistent and precise

LENGTH

Index of length lesson ideas

Length 1.1 lesson ideas

Identification of the attribute: make direct comparisons of length

Knowledge and strategies

1. use length vocabulary, e.g. *long*, *high*, *tall*; *short* or *low*; *the same as*
2. put two lengths side by side to compare their lengths
3. straighten a curved or bent length to check if two lengths are the same

Who is tall, who is short?

Students choose a classmate to stand beside. Students compare their heights by looking in a mirror or by asking another pair of students to assist. "Who is tall?" "Who is short?" Students draw a picture of themselves with their partner and label the two figures as "tall" and "short".

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.5	paper and pencil for recording, mirror	1. use length vocabulary, e.g. *long*, *high*, *tall*; *short* or *low*; *the same as* 2. put two lengths side by side to compare their lengths

Am I taller or shorter?

Students move independently around the classroom and identify three objects that are taller than or shorter than themselves. Record by drawing and labelling.

Alternatively, students choose a referent such as their desk and find three things that are shorter than, longer than, higher than their desk.

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.2 WMES1.5	classroom furniture, paper and pencil for recording	1. use length vocabulary, e.g. *long*, *high*, *tall*; *short* or *low*; *the same as* 2. put two lengths side by side to compare their lengths

Short and long paths and towers

Students use wooden blocks to make paths around the classroom and discuss whether they have made short paths or long paths.

Variations: make towers of different heights, snakes of different lengths, paint or draw long and short lines.

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.2	enough wooden blocks for each pair of students to make several paths or towers, chalk, paint, plasticine or play dough	1. use length vocabulary, e.g. *long*, *high*, *tall*; *short* or *low*; *the same as* 2. put two lengths side by side to compare their lengths

Longer than, shorter than our string (see lesson plan)

Students in pairs cut a piece of string and then move around the classroom to find as many objects as they can that are the same length as, longer than or shorter than the string. Students record their findings by drawing and labelling.

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.2	streamers or balls of string, scissors, paper and pencil for recording	1. use length vocabulary, e.g. *long*, *high*, *tall*; *short* or *low*; *the same as* 2. put two lengths side by side to compare their lengths 3. straighten a curved or bent length to check if two lengths are the same

Sort me!

Given a collection of lengths of braid, streamers, cardboard strips or ribbon, students sort them into a *long* pile and a *short* pile.

Students record their lengths and label as *long* and *short*.

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.2	braid, cardboard strips, ribbon or streamers cut to obviously long and short lengths, paper and pencil for recording	1. use length vocabulary, e.g. *long*, *high*, *tall*; *short* or *low*; *the same as* 2. put two lengths side by side to compare their lengths 3. straighten a curved or bent length to check if two lengths are the same

Length 1.1 lesson plan

Identification of the attribute: make direct comparisons of length

Longer than, shorter than our string

Students in pairs cut a piece of string or streamer and then move around the classroom to find as many objects as they can that are longer than or shorter than the string. Students record their findings by drawing and labelling.

Students should:

1. use length vocabulary, e.g. *long*, *high*, *tall*; *short* or *low*; *the same as*
2. put two lengths side by side to compare their lengths
3. straighten a curved or bent length to check if two lengths are the same.

Grouping

Step 1: whole-class discussion

Step 2: students find partner with matching straw

Step 3: whole-class discussion

Outcomes

MES1.1 Describes length and distance using everyday language and compares lengths using direct comparison.

WMES1.2 Uses objects, actions, imagery, technology and/or trial and error to explore mathematical problems.

Materials:

streamers or balls of string, scissors, paper, pencils for recording and sticks for demonstrating long and short

LENGTH

Activity

Step 1

- Introduce the lesson as a length activity, or by talking about *long*, *short* or *the same length as*.
- Discuss how objects can be described as being long or short.
- Display two sticks which are obviously long and short.
- Discuss how to compare length, i.e. hold the sticks side by side.
- Model how to use terms *longer*, *shorter*, *the same length as*.
- Introduce the task. Demonstrate how to record, if necessary.

Questioning

What can you tell me about these two sticks?
What are the special words we are using?
How do you work out which is long or short?
Which one is short, long?
How would we say that?

Step 2

- Students in pairs cut their string or streamer and then move around the room to compare their length with other objects.

Check that students

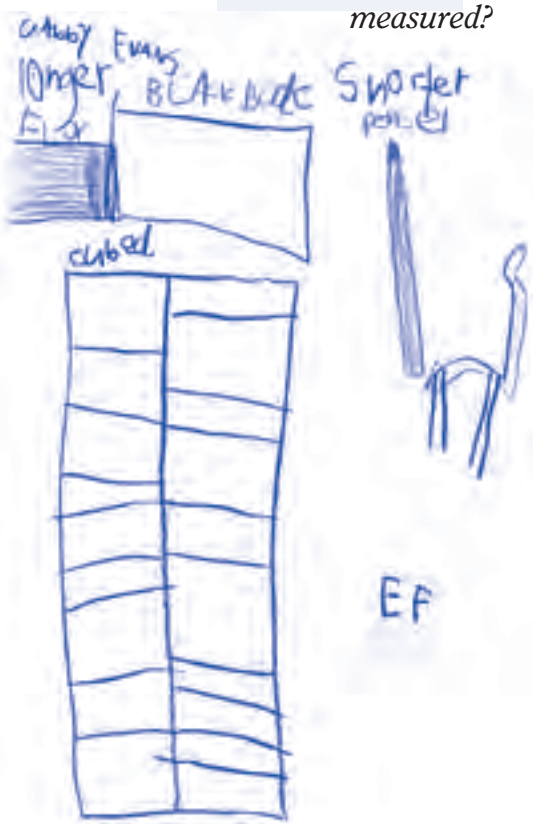
- are using terminology *long*, *short*, *the same as*
- place two lengths side by side.

Step 3

- Revise the terminology used to describe length.
- Ask students to report on what they found.

Discussion

What are the words we use to talk about length?
What did you find out when you measured?



Length 1.2 lesson ideas

Identification of the attribute: order two or more lengths by direct comparison

Knowledge and strategies

1. use comparative language, e.g. *longer*, *higher*, *taller than*; *shorter* or *lower than*; *the same as*; *longest*, shortest
2. ensure that ends are aligned for comparison by establishing a baseline
3. compare lengths systematically and explain why a length fits into a particular ordering

Stand in order

Small groups of students are ordered from tallest to shortest, e.g. students waiting to tell news, students who are wearing shorts, students who have an apple for lunch.

Use terminology *shorter than*, *taller than*, *shortest* and *tallest*.

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.2 WMES1.5	none	1. use comparative language, e.g. *longer*, *higher*, *taller than*; *shorter* or *lower than*; *the same as*; *longest*, *shortest* 2. ensure that ends are aligned for comparison by establishing a baseline 3. compare lengths systematically and explain why a length fits into a particular ordering

Straws in order

Given a number of straws of different lengths, students put them in order from longest to shortest. Straws are used because they will not stand up so students have to decide which end will be the baseline. These ends of the straws should be together.

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.2	straws of different lengths	1. use comparative language, e.g. *longer*, *higher*, *taller than*; *shorter* or *lower than*; *the same as*; *longest*, *shortest* 2. ensure that ends are aligned for comparison by establishing a baseline 3. compare lengths systematically and explain why a length fits into a particular ordering

Cutting equal lengths

Students cut a length of string or streamer and then use this to cut a second piece the same length. Compare the lengths with others in the group and order from the shortest to the longest.

Note: if the streamers or string tend to curl or kink, it may be necessary to sticky tape these to the desk to compare the lengths.

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.2	strings or streamers, scissors, tape	1. use comparative language, e.g. *longer*, *higher*, *taller than*; *shorter* or *lower than*; *the same as*; *longest*, *shortest* 2. ensure that ends are aligned for comparison by establishing a baseline 3. compare lengths systematically and explain why a length fits into a particular ordering

Order the group (see lesson plan)

Order from longest to shortest, three or more lengths which students have to straighten out and lay side by side, e.g. a skipping rope, a length of string and a rolled-up streamer. Record and label the lengths as *longest* and *shortest*. Report the results using comparative language.

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.4	a collection of lengths (rope, braid, string, ribbon, streamer, wool), recording paper.	1. use comparative language, e.g. *longer*, *higher*, *taller than*; *shorter* or *lower than*; *the same as*; *longest*, *shortest* 2. ensure that ends are aligned for comparison by establishing a baseline 3. compare lengths systematically and explain why a length fits into a particular ordering

Gorilla arms

Students in pairs or a small group order the group members by length of outstretched arms. Compare arms in pairs by matching fingertips on one side as a baseline. Ensure that all members of the group have been matched, to find the order. Record and label.

Outcomes	Materials	Knowledge and strategies
MES1.1 WMES1.2 WMES1.5	students, recording paper and pencils	1. use comparative language, e.g. *longer*, *higher*, *taller than*; *shorter* or *lower than*; *the same as*; *longest*, *shortest* 2. ensure that ends are aligned for comparison by establishing a baseline 3. compare lengths systematically and explain why a length fits into a particular ordering

Length 1.2 lesson plan

Identification of the attribute: order two or more lengths by direct comparison

Order the group

Order from longest to shortest, three or more lengths which students have to straighten out and lay side by side, e.g. a skipping rope, a length of string or a rolled-up streamer. Report the results using comparative language.

Students should:

1. use comparative language, e.g. *longer*, *higher*, *taller than*; *shorter* or *lower than*; *the same as*; *longest*, *shortest*
2. ensure that ends are aligned for comparison by establishing a baseline
3. compare lengths systematically and explain why a length fits into a particular ordering

Outcomes

MES1.1 Describes length and distance using everyday language and compares lengths using direct comparison.

WMES1.4 Uses concrete materials and/or pictorial representations to support conclusions.

Grouping

Step 1: whole-class discussion

Step 2: work in pairs or small groups

Step 3: whole-class discussion.

Materials:

a collection of three or more lengths for each pair or small group (e.g. rope, braid, string, ribbon, streamer, wool), recording paper.

LENGTH

Activity

Step 1

- Introduce the activity as the measurement of length.
- Class discussion of how to find and compare longer and shorter items.
- Introduce the task.

Questioning

How could I find a pencil longer than this one? What are the length words I could use? (long, short, longer than, shorter than)
If I had three pencils, how would I put them into order by length?

What words might I need to use? (longest, shortest)

What will you have to be careful about when you put the materials into order? (straighten the lengths, use a consistent baseline)

Step 2

- Students work in pairs or small groups to order the lengths of material by straightening out and laying side by side.
- *Variation:* Students record by drawing or writing, or both.

Check that students:

- are able to describe their ordering
- use appropriate terminology
- demonstrate the use of a baseline.

Note: students may need terminology written on the chalkboard to assist with recording.

Step 3 Conclusion

- Report back to the teacher or to the whole-class.

Discussion

What did you learn to do today?

What do you call it when you make sure that all of your lengths start at the same position?

What does this report show or tell us?

Length 2.1 lesson ideas

Informal measurement: choose and use appropriate units for measuring a length (no gaps or overlaps)

Knowledge and strategies

1. align identical units end to end along the given line without overlapping or leaving gaps
2. state or record that the length is the number and type of units used
3. use approximate language to explain parts of units, "about half a unit"
4. measure a circumference using string or paper strips, without overlapping ends

Choose my unit

Students choose from a collection of different units the ones they will use to measure a line (drawn on the floor with chalk or narrow tape). Use the units to make a line the same length as the one they are measuring on the floor. It is essential that students choose and use a set of identical units.

Record which units were used and how the line was measured.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.2	selection of units: rods, straws, popsticks, connecting blocks (multiple copies of each unit should be readily available)	1. align identical units end to end along the given line without overlapping or leaving gaps 2. state or record that the length is the number and type of units used 3. use approximate language to explain parts of units, "about half a unit"

Making lengths

Make a length the same as one made by the teacher using units (e.g. popsticks, paperclips) and glued onto cardboard. Students place their units:

1. in a straight line
2. not in a straight line (e.g. a curved or zigzag line).

Students comment on which line may be shorter or longer.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.4	selection of units: rods, straws, popsticks, matchsticks, etc.	1. align identical units end to end along the given line without overlapping or leaving gaps 2. state or record that the length is the number and type of units used 3. use approximate language to explain parts of units, "about half a unit"

Alternatives

Use different units to measure the same length, for example, "I needed six straws or nine popsticks". Record the units used and how the length was measured.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.2	selection of units: rods, straws, popsticks, skewers, matchsticks, etc.	1. align identical units end to end along the given line without overlapping or leaving gaps 2. state or record that the length is the number and type of units used 3. use approximate language to explain parts of units, "about half a unit"

Which one?

Students are given a box of pieces of string or strips of paper. Students have to find the piece which is exactly five units long.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.1	container of strips of paper or string, one piece to be an exact length, units for measuring	1. align identical units end to end along the given line without overlapping or leaving gaps 2. state or record that the length is the number and type of units used 3. use approximate language to explain parts of units, "about half a unit"

Who has the biggest head? (see lesson plan)

Students measure around their heads with paper strips and mark correctly without overlap. Measure the length of the string in units, (rods, paperclips, etc.) to find who has the biggest head in their group. Record group measurements and the units used.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.3	string, scissors units: rods, straws, connecting blocks, paper and pencil for recording	1. align identical units end to end along the given line without overlapping or leaving gaps 2. state or record that the length is the number and type of units used 3. use approximate language to explain parts of units, "about half a unit" 4. measure a circumference using string or paper strips, without overlapping ends

Length 2.1 lesson plan

Informal measurement: choose and use appropriate units for measuring a length

Who has the biggest head?

Students measure around their heads with string or paper strips and mark correctly without overlap. Measure the length in units (rods, paperclips, etc.) to find who has the biggest head in the group. Record group measurements and the units used.

Extension

This activity may be extended by asking students to work in groups of four, and listing the four head measurements in order of size for a group report. All group members will need to use the same unit of measure.

Students should:

1. align identical units end to end along the given line without overlapping or leaving gaps
2. state or record that the length is the number and type of units used
3. use approximate language to explain parts of units, "about half a unit"
4. measure a circumference using string or paper strips, without overlapping ends.

Outcomes

MS1.1 Estimates, compares and records lengths and distances using informal units, metres and centimetres.

WMS1.3 Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols.

Grouping

Step 1: whole-class discussion

Step 2: work in pairs or small groups

Step 3: whole-class discussion.

Materials:

string, scissors, units: rods, straws, connecting blocks, pencil and paper for recording

LENGTH

Activity

Step 1

- Discuss the measurement of length and the terminology used.
- Discuss how to measure head size. Students may suggest and demonstrate some alternative methods which can be discussed by the class.
- Emphasise the skills of placing and counting the measuring units. (no gaps, no overlaps)
- Discuss what to do with fractional units.
- Display the names of the units, for students to use in recording.

Step 2

- Students assist each other to measure around heads.
- Students choose, align, count and record the number of units used to measure.

Step 3

- Report back to a small group, the teacher or the whole class.

Questioning

What are we measuring when we use the words long, short, longer than, shorter, the same length, shorter than?

How could we measure your heads?

What advice can you give to someone who is measuring with units?

How can we record the head measurements?

What are we measuring when we use the words long, short, longer than, shorter, the same length, shorter than?

How could we measure your heads?

What advice can you give to someone who is measuring with units?

How can we record the head measurements?

This may be an opportunity for individual assessment to check that students have:

- chosen one kind of unit
- aligned and counted units correctly.

Discussion

Which units were good to use, and why?

Can we compare head sizes, e.g. if Casey used blocks for units and Tim used matchsticks?

Length 2.2 lesson ideas

Informal measurement: compare and order lengths by using identical units for each length

Knowledge and strategies

1. choose identical units to measure lengths
2. know that the longer line has more units
3. estimate the number of units and explain the estimation strategy
4. know that the length is conserved if rearranged

Let's compare

Estimate, then measure the height (or width) of items that are in different parts of the classroom: which is higher, the teacher's chair or a student chair; which is wider, a student desk or the teacher's table; which is higher, a bookshelf or the tote tray holder. Students record results and report back to the class.

Outcomes

MS1.1
WMS1.2

Materials

classroom furniture, string, selection of units: rods, straws, popsticks, etc.

Knowledge and strategies

1. choose identical units to measure lengths
2. know that the longer line has more units
3. estimate the number of units and explain the estimation strategy

Measure me

Students in pairs take turns to lie on the floor or ground to be measured. Partners mark the body length at the top of the head and the heels and draw a straight line joining the two points. Students measure their length in units. They compare the length of different people and check by direct comparison.

Outcomes

MS1.1
WMS1.2

Materials

chalk, a selection of units: rods, straws, popsticks, etc.

Knowledge and strategies

1. choose identical units to measure lengths
2. know that the longer line has more units
3. estimate the number of units and explain the estimation strategy

Does the tallest person have the longest feet? (see lesson plan)

Students work in groups of four to measure their foot lengths in units and record in order of length. They also record their heights in order, before comparing the two lists and writing an answer to the question.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.2	paper, pencils, a selection of units such as rods, straws, popsticks	1. choose identical units to measure lengths 2. know that the longer line has more units 3. estimate the number of units and explain the estimation strategy

Are they the same?

Give students two identical lengths of streamer. They paste one onto butchers' paper, then cut up the other. Are they still the same lengths? Can you prove they are the same length? Paste cut-up pieces onto a piece of paper, measure the two lengths with units. Discuss and report back.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.5	two lengths of streamer, butchers' paper, scissors, paste, pencil and paper	1. choose identical units to measure lengths 4. know that the length is conserved if rearranged

What's the trick?

Each student is given a container of units (popsticks, straws, rods) students make as many different lines as possible, each line 10 units long. Record the different arrangements and state if they are the same or different lengths.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.2	units, paper and pencil for recording	1. choose identical units to measure lengths 2. know that the longer line has more units 4. know that the length is conserved if rearranged

Length 2.2 lesson plan

Informal measurement: compare and order lengths by using identical units for each length

Does the tallest person have the longest feet?

Students work in groups of four to estimate then measure their foot lengths in units and record in order of length. They also record their heights in order, before comparing the two lists and writing an answer to the question.

Students should:

1. choose identical units to measure lengths
2. know that the longer line has more units
3. estimate the number of units and explain the estimation strategy

Grouping

Step 1: whole-class discussion

Step 2: work in groups of four

Step 3: whole-class discussion.

Outcomes

MS1.1 Estimates, compares and records lengths and distances using informal units, metres and centimetres.

WMS1.2 Uses objects, diagrams, imagery and technology to explore mathematical problems.

Materials:

paper, pencils, selection of units, activity sheet with the question: "Does the tallest person have the longest feet?"

Activity

Step 1

- Introduce the lesson as a measurement activity, focusing on length.
- Discuss the activity question and what the task will involve. Emphasise that the question should be answered "Yes" or "No", accompanied by an explanation and information.
- Ensure that students have access to enough units to measure lengths.

Questioning

What are we finding when we measure how long or short something is?

What will you have to measure to answer the activity question? What would be the easiest way to find the order of heights in your group?

What would the recording look like?

How will you measure the foot lengths in your group? What must all group members do, so that you can compare lengths?

Step 2

- Students work in groups to choose and use units.
- Students discuss measures so they can record names in order of foot length and height.

Check that students:

- align and count units correctly
- record names in order
- look for fractions of units, and count these correctly.

Step 3

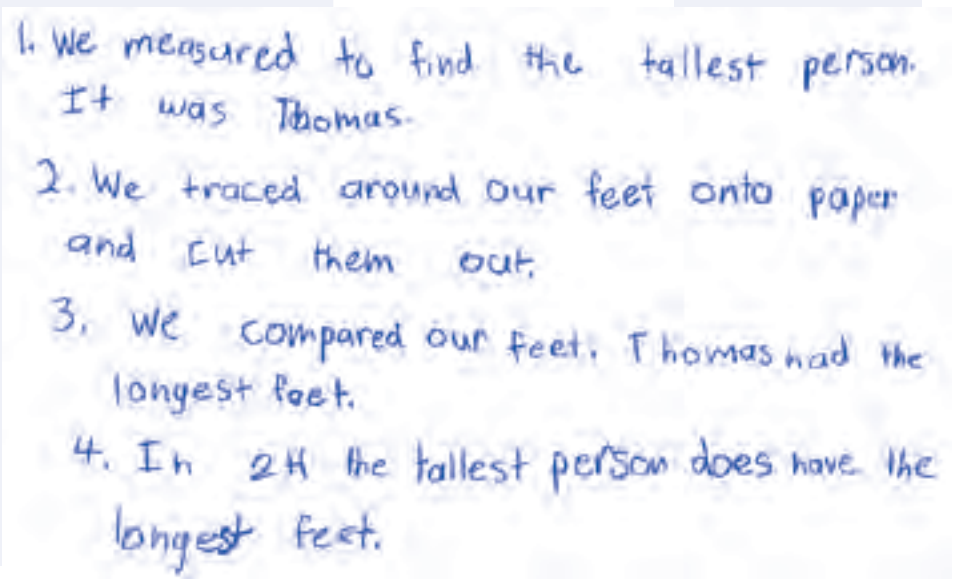
- Groups report back to the whole class.
- Compare methods of data display.
- Discuss whether the answers were predictable.

Discussion

What is your group's answer to the question?

How can you prove your answer? Which reports are easiest to read and interpret?

Are there other body measurements we can compare?


1. We measured to find the tallest person. It was Thomas.
2. We traced around our feet onto paper and cut them out.
3. We compared our feet. Thomas had the longest feet.
4. In 2H the tallest person does have the longest feet.

Length 3.1 lesson ideas

Structure of repeated units: use one unit to work out how many will be needed altogether when making indirect comparisons

Knowledge and strategies

1. measure precisely by repeating one unit
2. know that lengths (not marks or spaces) are counted
3. use a 10 cm strip as a unit to measure length

Measure with one unit

Students are given one specific unit (e.g. one popstick, streamer or 1 m ruler). Measure and compare objects in the classroom or playground, such as the circumference of the tree.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.2	units: rods, straws, popsticks, streamers, etc.	1. measure precisely by repeating one unit 2. know that lengths (not marks or spaces) are counted

Who wins?

One beetle walks around two sides of a desk or book, and her friend walks diagonally across the desk. Which beetle will walk the furthest? Vary this activity by choosing different routes.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.4	units: a paperclip, match etc., pencil and paper	1. measure precisely by repeating one unit 2. know that lengths (not marks or spaces) are counted

Make a ruler (see lesson plan)

Students make their own rulers based on an informal unit (teddybear, paperclip) or a body part (foot, hand span). Students should align units, especially teddybears or paperclips, end to end. Mark the scale on the ruler and use it to measure objects.

(This may need two lessons: make the ruler; use the ruler to measure objects.)

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.2	units: paperclips, matches, cardboard, teddybears, cardboard strips, pencil and paper	1. measure precisely by repeating one unit 2. know that lengths (not marks or spaces) are counted

Make a decimetre

Students work in pairs and make a ruler using 1 cm grid on light card. Cut out a 10 cm strip and colour one long edge as the measuring edge. Label the centimetre end points from 1 to 10.

Find and record objects which measure longer than 10 cm, less than 10 cm or longer than 100 cm.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.2	1 cm grid on light card, scissors, coloured pencils, pencils, paper	1. measure precisely by repeating one unit 2. know that lengths (not marks or spaces) are counted 3. use a 10 cm strip as a unit to measure length

Straw toss

Who can throw a straw the furthest; how much further is it than the next best throw?

Measure and record the distance thrown, using a 10 cm strip. Find the difference between the longest and shortest throw.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.2	straws, 10 cm strips, pencils, paper	1. measure precisely by repeating one unit 2. know that lengths (not marks or spaces) are counted 3. use a 10 cm strip as a unit to measure length

Length 3.1 lesson plan

Structure of repeated units: use one unit to work out how many will be needed altogether when making indirect comparisons

Make a ruler

Students make their own ruler based on an informal unit (teddybear, paperclip) or a body part (foot, hand span). Students should align units, especially teddybears or paperclips, end to end. Mark the scale on the ruler and use it to measure objects.

Students should:

1. measure precisely by repeating one unit
2. know that lengths (not marks) are counted

Grouping

Step 1: whole-class discussion

Step 2: individual or paired work

Step 3: whole-class reporting back.

Outcomes

MS1.1 Estimates, compares and records lengths and distances using informal units, metres and centimetres.

WMS1.2 Uses objects, diagrams, imagery and technology to explore mathematical problems.

Materials:

long strips of paper, pencils, small single items to be used as units of measure (paperclips, blocks, teddybears, cuisenaire rods)

LENGTH

Activity

Step 1

- Explain that the students will make a ruler, and then use the ruler to measure and compare the length or height of objects inside or outside the classroom.

Questioning

How can we compare the lengths of two objects without putting them side by side? What would you use?

What does a ruler look like? How do you know it is a ruler? What do the marks mean?

How could you make your own ruler?

How would you start?

What would make a good unit?

What lengths could you measure around this classroom?

Step 2

- Students work individually or in pairs to choose a unit and make a ruler.
- Use the ruler to measure objects. Encourage students to estimate first.
- Record the measurements.

Check that students:

- align and mark their units correctly (some students may not know where to write "1" on their ruler)
- start from the 0 point, when using their ruler to measure
- record measurements correctly, e.g. 15 bears
- calculate the total length correctly when an object is longer that one ruler-length.

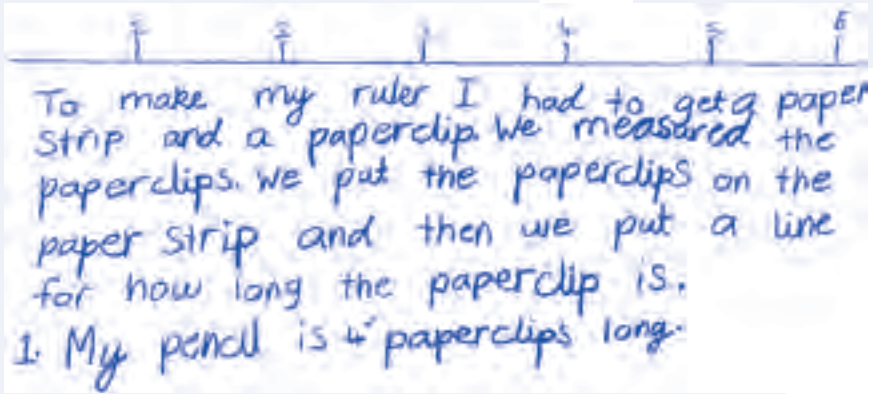
Step 3

- Whole-class discussion and demonstration of how rulers were made and used.
- Discuss what will happen when two different rulers are used to measure the same object.

Discussion

What advice would you give about making and using your own ruler?

Why do we have different measurements for this same length?



Length 3.2 lesson ideas

Structure of repeated units: explain the relationship between unit size and the number of units used to measure length

Knowledge and strategies

1. explain the relationship between unit size and number of units
2. use the metre as a unit to measure lengths
3. know that measurement techniques must be consistent and precise

What's the difference? (see lesson plan)

Students use different units to measure the same object and are asked to explain why the measurements are different (e.g. Athena measured the book as 5 rods but Alex measured it as 15 paperclips). Alternatively a 1 m ruler or 10 cm strip could be used.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.5	paper, pencils, items for use as units of measure (10 cm strips, 20 cm strips, blocks, cuisenaire rods, paperclips, popsticks etc.)	1. explain the relationship between unit size and number of units 2. use the metre as a unit to measure lengths 3. know that measurement techniques must be consistent and precise

Body parts

Students working in groups of three or four use a body length (or hand span or the length of an arm or leg) as the unit of measure to find the width of the classroom. Students record their results and then explain why the measurements from several groups may be different. During a class discussion, students predict how many more or fewer units will be needed when the measurement is expressed in a different unit.

Outcomes	Materials	Knowledge and strategies
MS1.1 WMS1.3	paper strips or streamers for a measuring unit, pencils and paper for recording	1. explain the relationship between unit size and number of units 3. know that measurement techniques must be consistent and precise

Towering metres

Build a tower that is one metre high.

Check height with a metre ruler.

Record how the estimate was made, and the measured result.

Outcomes	Materials	K & S
MS1.1 WMS1.4	building objects for tower, metre rulers, paper and pencils	2. use the metre as a unit to measure lengths 3. know that measurement techniques must be consistent and precise

Ready, set, go!

Students work in small groups to estimate, then measure and record the process.

How long does it take to write and measure a legible sentence 1 metre long?

How long does it take to make and measure a line of pens (popsticks, match sticks) one metre long?

How long does it take to make and measure a modelling dough snake 1 metre long?

Outcomes	Materials	K & S
MS1.1 WMS1.4	watch, metre measure or metre ruler, paper, pencils, sticks, modelling dough	2. use the metre as a unit to measure lengths 3. know that measurement techniques must be consistent and precise

Snakes alive

Small group of students predict how many 100 gm lumps of plasticine or modelling dough will be used to make a one metre snake. Make snake and check length by measuring and marking in 10 cm lengths.

Record and report on how much plasticine was used, and how much would be necessary for five snakes.

Outcomes	Materials	K & S
MS1.1 WMS1.4	plasticine or play dough, 100 gm mass, 10 cm measuring strip	2. use the metre as a unit to measure lengths 3. know that measurement techniques must be consistent and precise

Length 3.2 lesson plan

Structure of repeated units: explain the relationship between unit size and the number of units used to measure length

What's the difference?

Students use different units to measure the same object and are asked to explain why the measurements are different (e.g. Athena measured the book as 5 rods but Alex measured it as 15 paperclips).

Students should:

1. explain the relationship between unit size and number of units
3. know that measurement techniques must be consistent and precise

Grouping

Step 1: whole-class discussion

Step 2: individual or paired activity

Step 3: whole-class report back and discuss results.

Outcomes

MS1.1 Estimates, compares and records lengths and distances using informal units, metres and centimetres.

WMS1.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 1 content.

Materials:

paper, pencils, items for use as units of measure (10 cm strips, 20 cm strips, blocks, cuisenaire rods, paperclips, popsticks etc.)

LENGTH

Activity

Step 1

- Explain that students will measure the length of objects by using two different units of measure for each object.
- The measurements should be recorded, and an explanation given for why different measurements can be found for the same object.

Questioning

What kind of answers will I get if I measure the length of this line with popsticks units and then paperclip units? Why?

How would my measurement change if I used large bears as units and then small bears as units? Why?

How will you record your measurements?

Step 2

- Students work in pairs or individually to measure and record the same length several times, using different units each time.
- Students should give a brief explanation of the difference(s) in measurement.

Check that students:

- measure consistently and accurately
- record measurements correctly
- estimate before measuring.

Step 3

Whole-class discussions:

Why different measurements can be found, for the same object.

How measurements may differ, through inconsistent methods.

How the ratio of the size of the measuring units will affect the number of units needed to measure (e.g. the 20 cm strip is twice as long as 10 cm, so you will need half as many).

Discussion

Can you read out your recording and explain to us what you found?

Why are these measurements different?

If I measured this length with 8 pink rods, can you estimate how many red rods would measure the same length? Why?

LENGTH

Teaching measurement: Area

Area

Area, or the amount of surface, is a two-dimensional quantity and has to be identified as a property of a three-dimensional object. The three-dimensional nature of the object being measured may obscure the two-dimensional nature of area. For example, the surface of a student's desk or the floor can be measured by overlaying it with square units. However, students may think that they are measuring the size of the desk itself because the concept of a surface with length and breadth but no width is difficult to imagine. Students may also gain the impression that areas are horizontal or vertical flat surfaces because such surfaces are most commonly measured. Students are likely to measure the area of the top of their desk, but not the areas of its sides, underneath surface, or legs. The areas of these surfaces are usually not measured, nor are other hard to measure areas, such as curved or irregular surfaces.

Students are usually introduced to the concept of area by superimposing areas and later, by measuring areas with informal units. In covering activities, rectangular areas are used so that students develop an understanding of the structure of the unit covering (array) in area. Knowledge of array structure is important for an understanding of area measurement as it enables the area of a rectangle to be linked to the lengths of its sides, and is fundamental to an understanding of the formula for the area of a rectangle. The array structure also provides the basis for rectangular area to be calculated using multiplication.

When young students draw the covering of a rectangle with unit squares, some will draw each individual square while others will draw a combination of individual squares and lines. Some students will draw lines to represent rows and then mark off squares individually while others will draw an array using lines. In general, these different methods appear to mirror students' understanding of the array structure and indicate if students have constructed rows (and/or columns) as composite units. Thus, drawing the covering appears to be an effective way of focusing students' attention on the array structure. However, tracing may not help students, as some of them will be able to trace an accurate array, yet not understand its structure. Drawing or visualising accurate arrays suggests that students can represent covering a region with rectangular units, without gaps or overlap.

Level descriptions for area

Level 1	Knowledge and strategies
L1.1 Identification of the attribute Make direct comparisons of area	1. use area vocabulary: *surface*, *inside*, *outside*, *shape*, *area*, *boundary*, *large area*, *small area* 2. make closed shapes; indicate the space enclosed by the boundary 3. superimpose shapes to compare their size (large differences) 4. indicate the surface they are referring to
L1.2 Identification of the attribute Order two or more areas by direct comparison	1. use comparative language, e.g. *larger area*, *smaller area*, *largest area*, *smallest area*, *the same area as* 2. superimpose same shapes to compare them 3. compare areas systematically and explain how an area fits into a particular ordering
Level 2	**Knowledge and strategies**
L2.1 Informal measurement Choose and use appropriate units for measuring area	1. structure identical units in rows or columns (no gaps or overlaps) to cover an area 2. state or record that the area is the number and type of units used 3. use approximate language for parts of units "about half a tile". 4. choose appropriate units and explain why one shape is better than another to use as a covering tile
L.2.2 Informal measurement Compare and order areas by covering each area with identical units	1. choose identical units and cover each area 2. know that the larger area has more units 3. estimate the number of units and explain the estimation strategy 4. know that area is conserved if rearranged
Level 3	**Knowledge and strategies**
L.3.1 Structure of repeated units Use one unit to work out how many will be needed altogether when making indirect comparisons	1. move and align the unit in a systematic way to preserve size 2. represent rows and columns by extending or drawing lines (rectangular units) 3. explain and use the structure of rectangular unit tessellation
L3.2 Structure of repeated units Explain the relationship between unit size and number of units used to measure area	1. explain the relationship between unit size and the number of units 2. express the same area in terms of different-sized units 3. know that measurement techniques must be consistent and precise

AREA

Index of area lesson ideas

Area 1.1 lesson ideas

Identification of the attribute: make direct comparisons of area

Knowledge and strategies

1. use area vocabulary: *surface*, *inside*, *outside*, *shape*, *area*, *boundary*, *large area*, *small area*
2. make closed shapes; indicate the space enclosed by the boundary
3. superimpose shapes to compare their size (large differences)
4. indicate the surface they are referring to

Closed and open (see lesson plan)

Given lengths of string or ribbon, students make lines and then shapes with the material and then draw these. Students indicate the area of the shape. Whole class join several lengths to make a large area. Students measure the area, in bodies, by lying down within the shape.

Alternatives: Use computer software to draw closed and open shapes, and fill the shapes with colour. Make small open and closed shapes with pipe cleaners.

Outcomes	Materials	Knowledge and strategies
MES1.2 WMES1.2	lengths of string, ribbon or tape, pencils and recording paper	1. use area vocabulary: *surface*, *inside*, *outside*, *shape*, *area*, *boundary*, *large area*, *small area* 2. make closed shapes; indicate the space enclosed by the boundary 4. indicate the surface they are referring to

Tablecloths

Make tablecloths to cover their desks. Discuss whether a tablecloth has the same area as their desk or whether it is larger or smaller in area.

Outcomes	Materials	Knowledge and strategies
MES1.2 WMES1.2 WMES1.3	butcher's paper, paint and scissors, tape	1. use area vocabulary: *surface*, *inside*, *outside*, *shape*, *area*, *boundary*, *large area*, *small area* 3. superimpose shapes to compare their size (large differences) 4. indicate the surface they are referring to

Cover me up

Students draw a large shape, paint or colour the area and cut it out. Compare the size of their shape with a partner's by superimposing.

Outcomes

MES1.2
WMES1.2
WMES1.3

Materials

paint, crayons, paper, scissors

Knowledge and strategies

1. use area vocabulary: *surface*, *inside*, *outside*, *shape*, *area*, *boundary*, *large area*, *small area*
2. make closed shapes; indicate the space enclosed by the boundary
3. superimpose shapes to compare their size (large differences)
4. indicate the surface they are referring to

Find a bigger area

Students are given a piece of paper and they have to find three areas that are bigger, smaller or about the same size. Teacher models how to compare the areas by superimposing.

Outcomes

MES1.2
WMES1.3

Materials

paper, classroom equipment

Knowledge and strategies

1. use area vocabulary: *surface*, *inside*, *outside*, *shape*, *area*, *boundary*, *large area*, *small area*
3. superimpose shapes to compare their size (large differences)
4. indicate the surface they are referring to

Match the shape

Students are given four small pieces of paper, card or fabric which are all the same shape but different sizes. Students select two shapes, compare by superimposing, and discuss which shape has the larger or smaller area.

Students are given a worksheet with the outlines of the four shapes in different orientations. The shapes are matched by superimposing and pasted down on the worksheet.

Outcomes

MES1.2
WMES1.2
WMES1.3

Materials

four shapes and a worksheet for each student, paste

Knowledge and strategies

1. use area vocabulary: *surface*, *inside*, *outside*, *shape*, *area*, *boundary*, *large area*, *small area*
3. superimpose shapes to compare their size (large differences)
4. indicate the surface they are referring to

Area 1.1 lesson plan

Identification of the attribute: make direct comparisons of area

Closed and open

Given lengths of string or ribbon, students make lines and then shapes with the material and then draw these. Students indicate the area of the shape. Whole class join several lengths to make a large area. Students measure the area, in bodies, by lying down within the shape.

Students should:

1. use area vocabulary: *surface*, *inside*, *outside*, *shape*, *area*, *boundary*, *large area*, *small area*
2. make closed shapes; indicate the space enclosed by the boundary
4. indicate the surface they are referring to.

Grouping

Step 1: whole class, then groups of 2 or 3

Step 2: whole class, then groups of 2 or 3

Step 3: whole class.

Outcomes

MES1.2 Describes area using everyday language and compares areas using direct comparison.

WMES1.2 Uses objects, imagery, actions, technology and/or trial and error to explore mathematical problems.

Materials:

lengths of string, ribbon or tape for each group of three students, pencil and recording paper

AREA

Activity

Step 1

- Introduce or revise the term *area*.
- In groups of three, students take a length of string or ribbon and experiment with lines on the floor or desks. Students record the different lines made. Lines may be straight, curved or zigzag.
- Report back for a whole class discussion, describing the lines.

Step 2

- Discuss how to make shapes with the length of material. Demonstrate the area of the shape enclosed by the length of material.
- Return to groups. Make a shape with the material. Students draw shapes and label if possible. Talk about the shapes and how they were made.

Step 3

- Whole class sit in a circle, several students make a shape from five or six joined braids on the floor in the middle of the circle.
- Students estimate how many children would be needed to cover the area of the shape by lying on the floor.
- Students direct volunteers to lie in the shape. Count the bodies.

Alternatives:

Use computer software to draw closed and open shapes, and fill the shapes with colour.

Make small open and closed shapes with pipe cleaners.

Questioning

"What is area?" Ask for explanations, give definition as the amount of surface.

Tell me about the lines which you are making. What does this line look like?

Check that students:

- can make shapes with their string
- know how to tie the ends of the string
- can indicate the area of the shape.

Discussion

What kind of shape have we got?

What could we cover the area of the shape with?

Can you estimate how many children we would need to cover this area if they were lying down?

Can they fit better if we put them another way?

How many children did we need to cover the area?

Are there any little spaces, not covered by bodies?

Area 1.2 lesson ideas

Identification of the attribute: order two or more areas by direct comparison

Knowledge and strategies

1. use comparative language, e.g. *larger area*, *smaller area*, *largest area*, *smallest area*, *the same area as*
2. superimpose same shapes to compare them
3. compare areas systematically and explain how an area fits into a particular ordering

Shoe-prints (see lesson plan)

Students work in small groups to trace each student's shoe on paper and cut these out. They superimpose the shoe-prints to find who has the largest or smallest shoe-print and explain how they found out. Place the shoe-prints in order by comparing the areas.

Note: Check that students do not confuse the longest shoe with the shoe which has the greatest area.

Outcomes	Materials	Knowledge and strategies
MES1.2 WMES1.3	paper, scissors, pencils	1. use comparative language, e.g. *larger area*, *smaller area*, *largest area*, *smallest area*, *the same area as* 2. superimpose same shapes to compare them 3. compare areas systematically and explain how an area fits into a particular ordering

Let's compare

Students take shapes from a bag and superimpose them to find out which has the largest or smallest area, then order the areas.

Outcomes	Materials	Knowledge and strategies
MES1.2 WMES1.2 WMES1.3	different enlargements of shapes, leaves, pieces of cloth	1. use comparative language, e.g. *larger area*, *smaller area*, *largest area*, *smallest area*, *the same area as* 2. superimpose same shapes to compare them 3. compare areas systematically and explain how an area fits into a particular ordering

Whose handprint?

A small group of students is given a copied handprint. Students have to make their own handprint, cut it out and compare it with the copied print to find whether their own is a larger or smaller area or about the same area. Superimpose the handprints and paste on top of each other to order the area.

Outcomes	Materials	Knowledge and strategies
MES1.2 WMES1.2 WMES1.3	copied handprint, paper, pencils, scissors	1. use comparative language, e.g. *larger area*, *smaller area*, *largest area*, *smallest area*, *the same area as* 2. superimpose same shapes to compare them 3. compare areas systematically and explain how an area fits into a particular ordering

Order the faces

Each student brings in a box from home, (e.g. cereal, tissue, toothpaste, foodwrap).

Students select and paint one of the faces to compare the area of the face from the box they have chosen.

In small groups of four, students compare and order the areas of the selected faces. Write or draw how they ordered the faces.

Outcomes	Materials	Knowledge and strategies
MES1.2 WMES1.3	variety of boxes (some with identical faces) or pieces of paper or fabric, pencil and paper	1. use comparative language, e.g. *larger area*, *smaller area*, *largest area*, *smallest area*, *the same area as* 2. superimpose same shapes to compare them 3. compare areas systematically and explain how an area fits into a particular ordering

Find a smaller face

Students are each given a small rectangle or square (approximately 15 cm x 10 cm). Students find three other objects in the classroom with a smaller area than their shape. Students superimpose their shape to find objects.

Record items found and place in order of size.

Outcomes	Materials	Knowledge and strategies
MES1.2 WMES1.3	rectangle or square for each student (different sizes) pencil and paper for recording	1. use comparative language, e.g. *larger area*, *smaller area*, *largest area*, *smallest area*, *the same area as* 2. superimpose same shapes to compare them 3. compare areas systematically and explain how an area fits into a particular ordering

Area 1.2 lesson plan

Identification of the attribute: order two or more areas by direct comparison

Shoe-prints

Students work in small groups to trace each student's shoe on paper and cut these out. They superimpose the shoe-prints to find who has the largest or smallest shoe-print and explain how they found out. Place the footprints in order by comparing the area.

Note: Check that students do not confuse the longest shoe with the shoe-print which has the greatest area.

Students should:

1. use comparative language, e.g. *larger area*, *smaller area*, *largest area*, *smallest area*, *the same area as*
2. superimpose same shapes to compare them
3. compare areas systematically and explain how an area fits into a particular ordering.

Grouping:

Step 1: whole-class discussion

Step 2: groups of four

Step 3: whole-class discussion.

Outcomes

MES1.2 Describes area using everyday language and compares areas using direct comparison.

WMES1.3 Describes mathematical situations using everyday language, actions, materials and informal recordings.

Materials:

paper, pencils, scissors

AREA

Activity

Step 1

- Revise the term *area*.
- Discuss how to find who has the largest shoe-print in the class.
- Suggested steps: find who has the largest shoe-print in a group of four, and then compare these as a whole class.

Questioning

"What is area?" Ask for explanations, give the definition as the amount of surface.

What do we call the mark you make when you walk on the floor with wet feet?

What would a shoe-print look like?

How can we find who has the largest shoe-print in the class?

What do we call the measuring technique when we place one area on top of another?

Step 2

- Students work with a partner to trace shoe. Cut it out.
- Compare the shoe-prints in the group by superimposing.
- Identify the largest shoe-print in the group.

Variation

Students record names of group members in order of area of shoe-prints.

Check that students:

- understand that they must cut carefully, so that part of the area will not be "lost"
- can superimpose two shoe-prints and judge which one has the larger area
- can point to and talk about and the larger area in their own language.

Step 3

- Whole class observing while students with largest shoe-prints from each group, compare their prints one at a time, to find the class largest.

Variation

Students find who has the largest handprint.

Discussion

What were you measuring?

How did you find the largest area?

Is there another body part that we could easily trace to compare the area?

Area 2.1 lesson ideas

Informal measurement: choose and use appropriate units for measuring area

Knowledge and strategies

1. structure identical units in rows or columns (no gaps or overlaps) to cover an area
2. state or record that the area is the number and type of units used
3. use approximate language for parts of units "about half a tile"
4. choose appropriate units and explain why one shape is better than another to use as a covering tile

How will I cover?

Students choose units (tessellating and non-tessellating) from a collection to cover a shape. Discuss the suitability of the units, in terms of no gaps or overlaps. Draw and describe how the shape was covered.

Outcomes

MS1.2
WMS1.1
WMS1.2

Materials

a variety of units: squares, circles, rectangles; shapes to be covered

Knowledge and strategies

1. structure identical units in rows or columns (no gaps or overlaps) to cover an area
2. state or record that the area is the number and type of units used
3. use approximate language for parts of units "about half a tile"
4. choose appropriate units and explain why one shape is better than another to use as a covering tile

Which is best?

Pairs of students select a unit and use multiple copies of the unit to cover different shapes. Pairs may decide to cover some shapes incorrectly by leaving gaps or overlaps, or by omitting rows of units.

Join with another pair, to discuss how the area was covered.

Outcomes

MS1.2
WMS1.1
WMS1.4

Materials

a variety of cardboard shapes and units

Knowledge and strategies

1. structure identical units in rows or columns (no gaps or overlaps) to cover an area
2. state or record that the area is the number and type of units used
3. use approximate language for parts of units "about half a tile"
4. choose appropriate units and explain why one shape is better than another to use as a covering tile

Make me! (see lesson plan)

Students make areas of a specific size, e.g. make a rectangle that has an area of 12 square units.

As each shape is made, students draw the pattern of units on paper.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS 1.3	a variety of square units, grid paper and pencils	1. structure identical units in rows or columns (no gaps or overlaps) to cover an area 2. state or record that the area is the number and type of units used

Roll the die twice

Student A rolls the die to find how many tiles to put along the top row. Student B rolls the die again to find how many rows to make. Predict how many tiles will be needed, after the second throw. Make the array, count the tiles, draw the pattern on grid paper.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.3	lots of small square tiles or bread tags, grid paper and pencils	1. structure identical units in rows or columns (no gaps or overlaps) to cover an area 2. state or record that the area is the number and type of units used

Cover up

Students tessellate rectangular units made from paper to cover a given shape. Paper units are used so that the students have to think carefully about the alignment of the units, whereas wooden or cardboard units fit together easily.

Students draw the pattern of the repeated units.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.1	a variety of different rectangular units made from paper, cardboard or drawn shapes, pencils and paper	1. structure identical units in rows or columns (no gaps or overlaps) to cover an area 2. state or record that the area is the number and type of units used 3. use approximate language for parts of units “about half a tile” 4. choose appropriate units and explain why one shape is better than another to use as a covering tile

Area 2.1 lesson plan

Informal measurement: choose and use appropriate units for measuring area

Make me!

Students make areas of a specific size, e.g. make a shape that has an area of 12 unit squares. As each shape is made, students draw the pattern of units on paper.

Students should:

1. structure identical units in rows or columns (no gaps or overlaps) to cover an area
2. state or record that the area is the number and type of units used

Grouping:

Step 1: whole-class introduction

Step 2: working in groups of 2 or 3

Step 3: whole-class reporting back and discussion.

Outcomes

MS1.2 Estimates, measures, compares and records areas using informal units.

WMS1.3 Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols.

Materials:

a variety of square units, grid paper and pencils

AREA

Activity

Step 1

- Revise how to measure length and area.
- Ask the students to make shapes which have an area of 12 units.
- Encourage students to think of as many different solutions as possible.

Step 2

- Students work with one or two partners to choose 12 tiles and make shapes.
- Record the shapes made and the pattern of the iteration.

Extension

- Students choose a second lot of 12 tiles (smaller or larger) to make more shapes with an area of 12 tiles.

Step 3

- Discuss whether the different shapes with the same number of tiles had different areas.

Extension

- Record how the size of the units affected the overall size of the shape made.

Questioning

What could I measure on this rectangular book or cardboard shape? (length of sides, area) What does "area" mean?

If I asked you to make a shape with an area of 12 square tiles, what kind of shape could I mean?

Check that students:

- use 12 tiles of the same size
- are able to talk about their shapes in terms of rows and columns
- remember "no gaps, no overlaps".

Discussion

When you cover a surface with tiles, what tile shapes are best to work with? Why? Would other shapes work?

These two shapes look different. They both have similar 12 tiles. Which one has a larger area?

Can you guess how my shape would be different if I chose 12 larger tiles? Why?

Area 2.2 lesson ideas

Informal measurement: compare and order areas by covering each area with identical units

Knowledge and strategies

1. choose identical units and cover each area
2. know that the larger area has more units
3. estimate the number of units and explain the estimation strategy
4. know that area is conserved if rearranged

Stick construction (see lesson plan)

Make shapes with 12 sticks (of equal length). Find which shape has the largest area by covering with tiles (the sides of the tiles must be the same length as the sticks). Variation: Make shapes on geoboards with elastic bands. Record shapes and their areas.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.2	tiles for measuring area, matchsticks or cut straws, geoboards, elastic bands, gridpaper	1. choose identical units and cover each area 2. know that the larger area has more units 3. estimate the number of units and explain the estimation strategy

Which is larger?

Given two shapes on opposite sides of a piece of paper (so they cannot be directly compared) students have to work out which figure is larger. Record the number of units used to measure each shape and draw the array pattern of repeated units.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.2	tiles for measuring area, shapes on opposite sides of a piece of paper or cardboard, paper and pencil	1. choose identical units and cover each area 2. know that the larger area has more units 3. estimate the number of units and explain the estimation strategy 4. know that area is conserved if rearranged

The same but different

Activity 1: Cover a shape with units. Then make another area that is identical in size but is a different shape. Explain why the two areas are identical.

Activity 2: Students are given a rectangle with marked grid lines. Students count and record the area in squares. Cut and rearrange the rectangle. Record the area of the new shape and explain why it is the same.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.3	tiles for measuring area, cardboard shapes, copied shape with marked gridlines, scissors, paper, paste	1. choose identical units and cover each area 2. know that the larger area has more units 3. estimate the number of units and explain the estimation strategy 4. know that area is conserved if rearranged

Make a jigsaw

Students are given identical pieces of paper (e.g. coloured squares). They draw a picture on the paper (to assist with re-assembling) and then cut the paper into 6, 9, or 12 pieces. In small groups discuss if their puzzles have stayed the same area and why.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.3	identical pieces of paper or light card, scissors, pencil and paper	4. know that area is conserved if rearranged

Don't lose the little bits which you cut off

Students are given two shapes which have a similar area, (e.g. a white triangle and a blue circle) and compare the areas by superimposing, cutting protruding parts and pasting one shape on top of the other. Students trial, then discuss which shape should be cut first. Students record which shape is the larger area, and why.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.2	two shapes to be superimposed and cut; scissors, paste, pencil and paper	4. know that area is conserved if rearranged

Area 2.2 lesson plan

Informal measurement: compare and order areas by covering each area with identical units

Stick construction

Make shapes with 12 sticks (of equal length). Find which shape has the largest area by covering with tiles (the sides of the tiles must be the same length as the sticks). Variation: Make shapes on geoboards with elastic bands. Record shapes and their areas.

Students should:

1. choose identical units and cover each area
2. know that the larger area has more units
3. estimate the number of units and explain the estimation strategy

Grouping:

Step 1 whole-class introduction

Step 2: pairs or small groups

Step 3: report back to whole class.

Outcomes

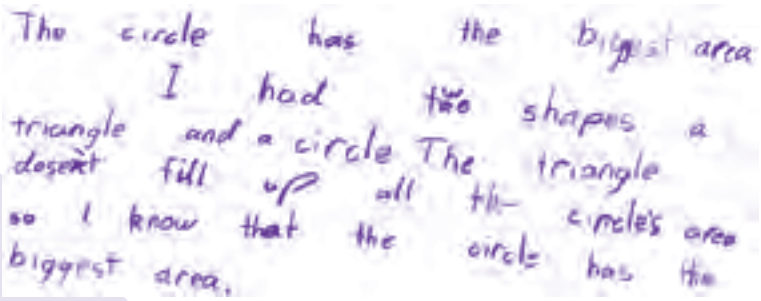
MS1.2 Estimates, measures, compares and records areas using informal units.

WMS1.2 Uses objects, diagrams, imagery and technology to explore mathematical problems.

Materials:

tiles for measuring area, matchsticks or cut straws, geoboards, elastic bands, grid paper, pencils and paper

AREA



Activity

Step 1

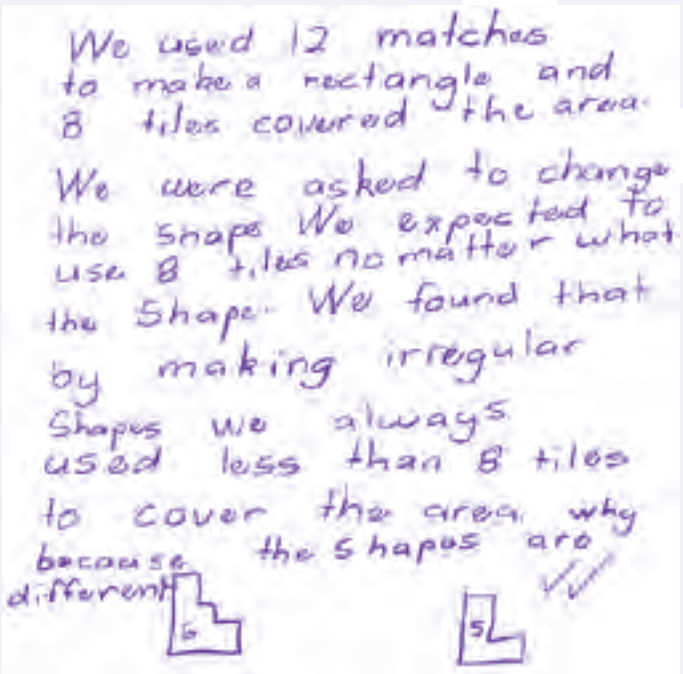
- Introduce the activity as the measurement of area.
- Ask students to experiment with making shapes with the 12 sticks.
- Shapes should have an area that can be measured with square tiles. The use of paper tiles may encourage students to fold some tiles, to produce half or quarter tiles.

Step 2

- Students should have access to as many tiles as they need to cover the shapes.
- Students work with their partner or group to make a variety of shapes.
- Estimate and measure the areas and record.

Step 3

- Discuss the variety of shapes made and their areas.



Questioning

If I make a rectangular shape with 12 sticks, what parts of the shape could I measure?

What kinds of shapes could I make with 12 sticks?

What do you have to remember to do when you measure area with tiles?

What will your recording look like?

What do you need to show?

Check that students:

- have chosen identical units and re-use these for each shape
- are using skip counting or stress counting strategies
- understand how to measure the area of each shape by using the units (some units may be folded to make partial units if students understand how to count the total number used).

Discussion

Which shape had the largest or smallest area?

Did all of your shapes have square corners? How did you measure the areas of shapes that didn't have square corners?

Area 3.1 lesson ideas

Structure of repeated units: use one unit to work out how many will be needed altogether when making indirect comparisons

Knowledge and strategies

1. move and align the unit in a systematic way to preserve size
2. represent rows and columns by extending or drawing lines (rectangular units)
3. explain and use the structure of rectangular unit tessellation (e.g. three rows of four or count as three, six, nine, twelve)

Which is bigger? (see lesson plan)

Using a 10 cm x 10 cm tile, students compare the areas of two rectangles taped on the floor or cut from cardboard (the rectangles should not be movable as the students may superimpose them). Students record their measurements, the results of the comparison, and the pattern of repeated units.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.2	rectangles made from tape or cardboard, 10 cm tiles, pencils and paper	1. move and align the unit in a systematic way to preserve size 2. represent rows and columns by extending or drawing lines (rectangular units) 3. explain and use the structure of rectangular unit tessellation (e.g. three rows of four or count as three, six, nine, twelve)

How many more? (see worksheet)

Shown a rectangle partly covered with squares, (e.g. one row and one column), students work out how many more squares would be needed to completely cover it (encourage students to visualise rather than drawing lines). This activity encourages students to visualise the array structure, from the available clues.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.2	worksheet with one row and one column of squares covered	1. move and align the unit in a systematic way to preserve size 2. represent rows and columns by extending or drawing lines (rectangular units) 3. explain and use the structure of rectangular unit tessellation (e.g. three rows of four or count as three, six, nine, twelve)

Tiling the bathroom

Given a rectangle ("floor plan"), students have to work out how many tiles will cover it (given only one 10 cm tile). Students create the tessellation by tracing or marking. Record the tiling and measurement.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.2	rectangular cardboard shapes, cardboard tiles, pencils and paper	1. move and align the unit in a systematic way to preserve size 2. represent rows and columns by extending or drawing lines (rectangular units) 3. explain and use the structure of rectangular unit tessellation (e.g. three rows of four or count as three, six, nine, twelve)

How do I know? (see worksheet)

Place different rectangles on 1 cm grid paper. Students have to work out which shape has the largest area, without taking the rectangles off the grid paper. Students should explain their reasoning and counting strategies. Encourage students to visualise the array pattern, rather than drawing lines.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.4	rectangular cardboard shapes, 1 cm grid paper, pencils and paper	1. move and align the unit in a systematic way to preserve size 2. represent rows and columns by extending or drawing lines (rectangular units) 3. explain and use the structure of rectangular unit tessellation (e.g. three rows of four or count as three, six, nine, twelve)

Draw the square

Given a rectangular-shaped picture and a square unit, students have to work out how many squares will cover the picture. Students count or calculate the area and draw the array pattern. Check against a model of the shape covered with squares. Students explain any differences. This activity reinforces the structure of the rectangular array.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.2	square tiles for measuring area, rectangular pictures or shapes, pencils and paper	1. move and align the unit in a systematic way to preserve size 2. represent rows and columns by extending or drawing lines (rectangular units) 3. explain and use the structure of rectangular unit tessellation (e.g. three rows of four or count as three, six, nine, twelve)

Area 3.1 lesson plan

Structure of repeated units: use one unit to measure area

Which is bigger?

Using a 10 cm x 10 cm tile, students compare the areas of two rectangles taped on the floor or cut from cardboard (the rectangles should not be movable as the students may superimpose them). Students record their measurements, the results of the comparison and the pattern of repeated units.

Students should:

1. move and align the unit in a systematic way to preserve size
2. represent rows and columns by extending or drawing lines (rectangular units)
3. explain the structure of rectangular unit tessellation

Grouping:

Step 1: whole-class discussion

Step 2: students work in groups of 2 or 3

Step 3: report back to whole class or larger group.

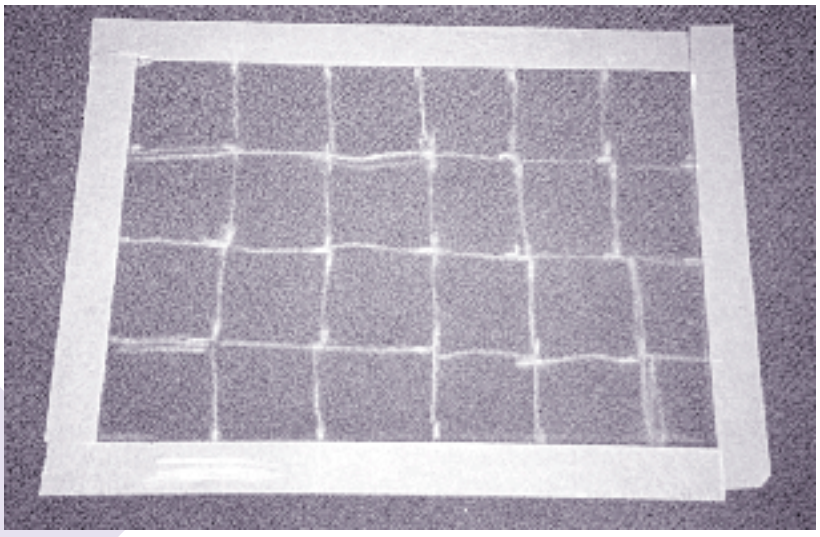
Outcomes

MS1.2 Estimates, measures, compares and records areas using informal units.

WMS1.2 Uses objects, diagrams, imagery and technology to explore mathematical problems.

Materials:

rectangles made from tape or cardboard, 10 cm tiles, pencils and paper



AREA

Activity

Step 1

- Introduce the activity as measuring which rectangle has the larger area.
- Discuss how area is measured by repeating one unit to make a row, and then repeating rows.
- Tell students that only one tile will be provided.
- Remind students that the measurements will be recorded.

Questioning

If you had two rectangles which could not be superimposed, how could you work out which has the larger area?

How would you measure the area? What could you use in our classroom?

What pattern would the tiles make? How would you draw this?

Is it possible to measure and compare the areas using only one tile?

How will you do this?

What will you need to show in your recording?

Step 2

- Students work with a partner to measure rectangles, record measurements and decide which rectangle is larger.
- Encourage students to draw the grid pattern of their measurements.

Check that students:

- know that the unit must be moved systematically, with marks made for the edges in two directions
- know that they are producing a grid pattern.

Step 3

- Discuss whether the tiles had to be moved and marked over the entire rectangles.

Discussion

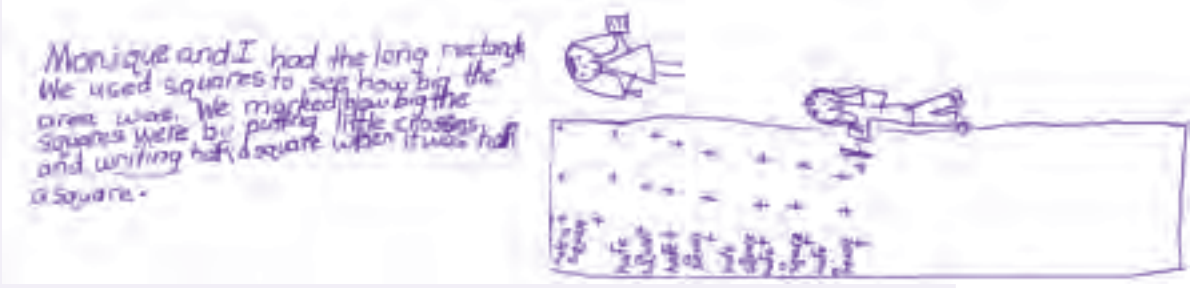
Was it necessary to keep moving your tile over the whole rectangle?

Can you explain this, and show us with your drawing?

What would happen if I chose a larger or smaller tile to measure the areas? Why?

Extension

Discuss how the measurements would differ if a larger or smaller tile had been chosen.



Area 3.2 lesson ideas

Structure of repeated units: explain the relationship between unit size and number of units used to measure area

Knowledge and strategies

1. explain the relationship between unit size and the number of units
2. express the same area in terms of different-sized units
3. know that measurement techniques must be consistent and precise

Large and small (see lesson plan)

Students are shown a rectangle covered with large squares and given some small squares (four to a large square). They are asked to work out how many small squares would cover the rectangle (or vice versa). Discuss which sized unit is better for measuring the area.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.5	rectangular cardboard shapes, two sizes of tiles (one a multiple of the other)	1. explain the relationship between unit size and the number of units 2. express the same area in terms of different-sized units 3. know that measurement techniques must be consistent and precise

Patchwork quilts

Emma made a patchwork quilt with 24 rectangles and Trent made one with 12 squares. Students work out if the quilts could be the same size (one square must equal two rectangles). The shapes may be changed but they must be multiples of each other so that students can make the quilts. Students should write and draw to explain their reasoning. *Suggestions*: twelve 10 cm squares and twenty-four 5 cm x 10 cm rectangles.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.3 WMS1.4	cardboard rectangles and tiles (one a multiple of the other)	1. explain the relationship between unit size and the number of units 2. express the same area in terms of different-sized units 3. know that measurement techniques must be consistent and precise

Hidden squares (see worksheet)

Students are shown a square grid with a large rectangular shape superimposed on it. The shape is covered in tiles that are an integral multiple of the units on the square grid. Students have to work out how many small units are hidden and record their working. (See worksheet)

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.2	worksheet tiles, pencils and paper	1. explain the relationship between unit size and the number of units 2. express the same area in terms of different-sized units 3. know that measurement techniques must be consistent and precise

Chessboard

Students are given a chessboard or similar grid. Their task is to make a tile so that they would need fewer tiles to cover the board, (e.g. if the grid had 24 squares could they make a cardboard tile consisting of 2, 4, 6, 8 or 12 squares, so that fewer would be needed to cover the board).

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.3 WMS1.4	grid, paper and scissors	1. explain the relationship between unit size and the number of units 2. express the same area in terms of different-sized units 3. know that measurement techniques must be consistent and precise

Magic tiles

Investigate the relationship between the areas of the hexagon, trapezium and triangle pattern blocks. Make larger hexagons, trapeziums and triangles using these blocks. Express the areas of the large shapes in terms of the blocks used, and then calculate the area in triangles.

Outcomes	Materials	Knowledge and strategies
MS1.2 WMS1.3 WMS1.4	pattern blocks, pencils and paper	1. explain the relationship between unit size and the number of units 2. express the same area in terms of different-sized units 3. know that measurement techniques must be consistent and precise

Area 3.2 lesson plan

Structure of repeated units: explain the relationship between unit size and number of units used to measure area

Large and small

Students are shown a rectangle covered with large squares and given some small squares (4 to a large square). They are asked to work out how many small squares would cover the rectangle (or vice versa). Discuss which sized unit is better for measuring the area.

Students should:

1. explain the relationship between unit size and the number of units
2. express the same area in terms of different-sized units
3. know that measurement techniques must be consistent and precise.

Grouping:

Step 1: whole-class introduction

Step 2: students work in pairs or small groups

Step 3: report back and discuss with class.

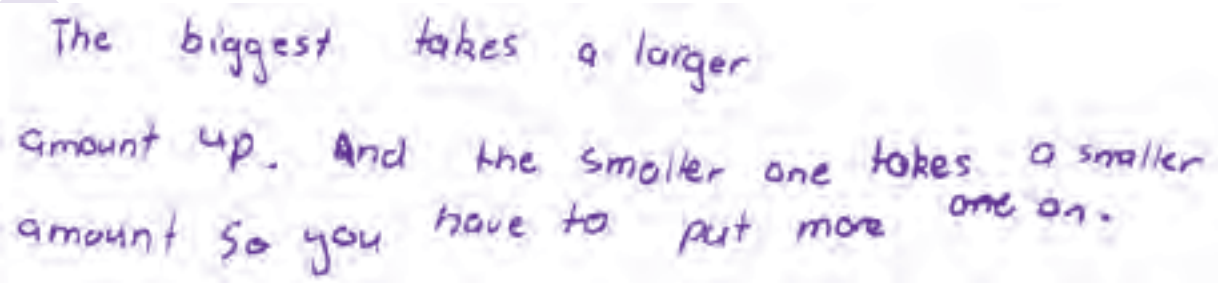
Outcomes

MS1.2 Estimates, measures, compares and records areas using informal units.

WMS1.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 1 content.

Materials:

rectangular cardboard shapes, two sizes of tiles (one a multiple of the other)



AREA

Activity

Step 1

- Introduce the lesson as working with *area* measurements.
- Explain that the area of the rectangle was measured with large square tiles, but we would also like to know how many small square tiles would be needed to cover the area.
- Students will be given one large tile and one small tile and will have access to the rectangle to be measured.

Step 2

- Students work with their partner to calculate the number of small tiles needed. Recording should explain clearly how calculations were made.

Step 3

- Discuss how to prove that the number calculated is correct.

Questioning

How would I measure the area of this shape?

If we said that this rectangle has an area of 12 square units, what does that mean?

What difference would it make if I used much smaller or bigger tiles for measuring? How do you know?

(Students working at this level should not need specific instructions of how to calculate and then check the number of tiles.)

Check that students:

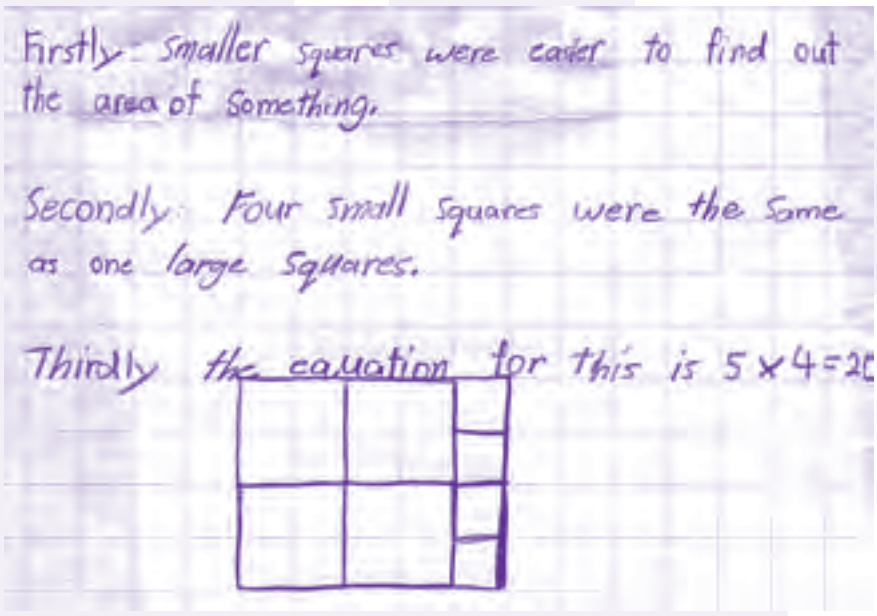
- are able to use different ways of calculating the total number of tiles
- are able to explain how they found an answer.

Discussion

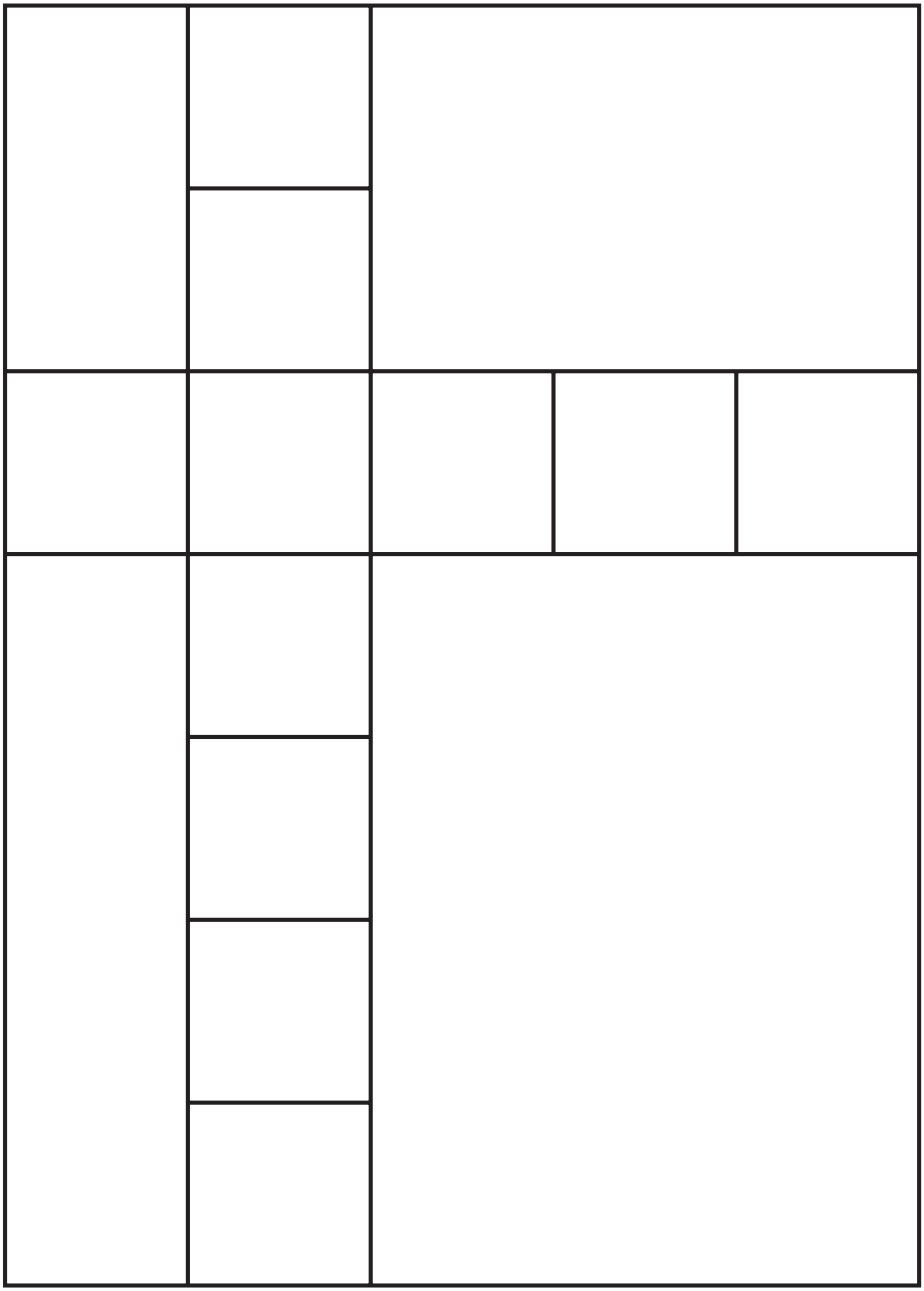
Are there different ways of working this out?

Variation:

Can you explain why this works with hexagons and triangles?



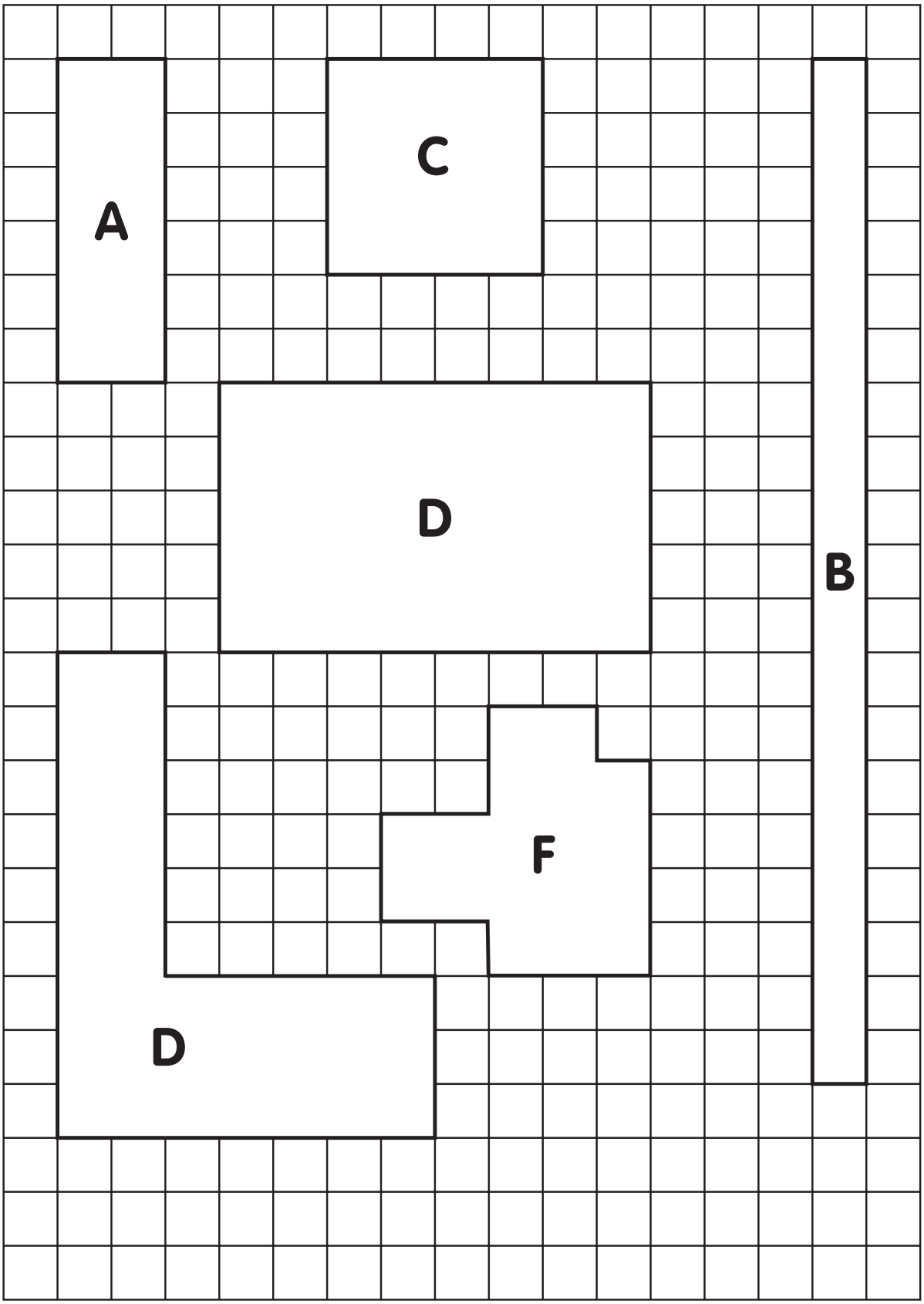
Worksheet for 3.1: How many more?



Worksheet

Worksheet for 3.1: How do I know?

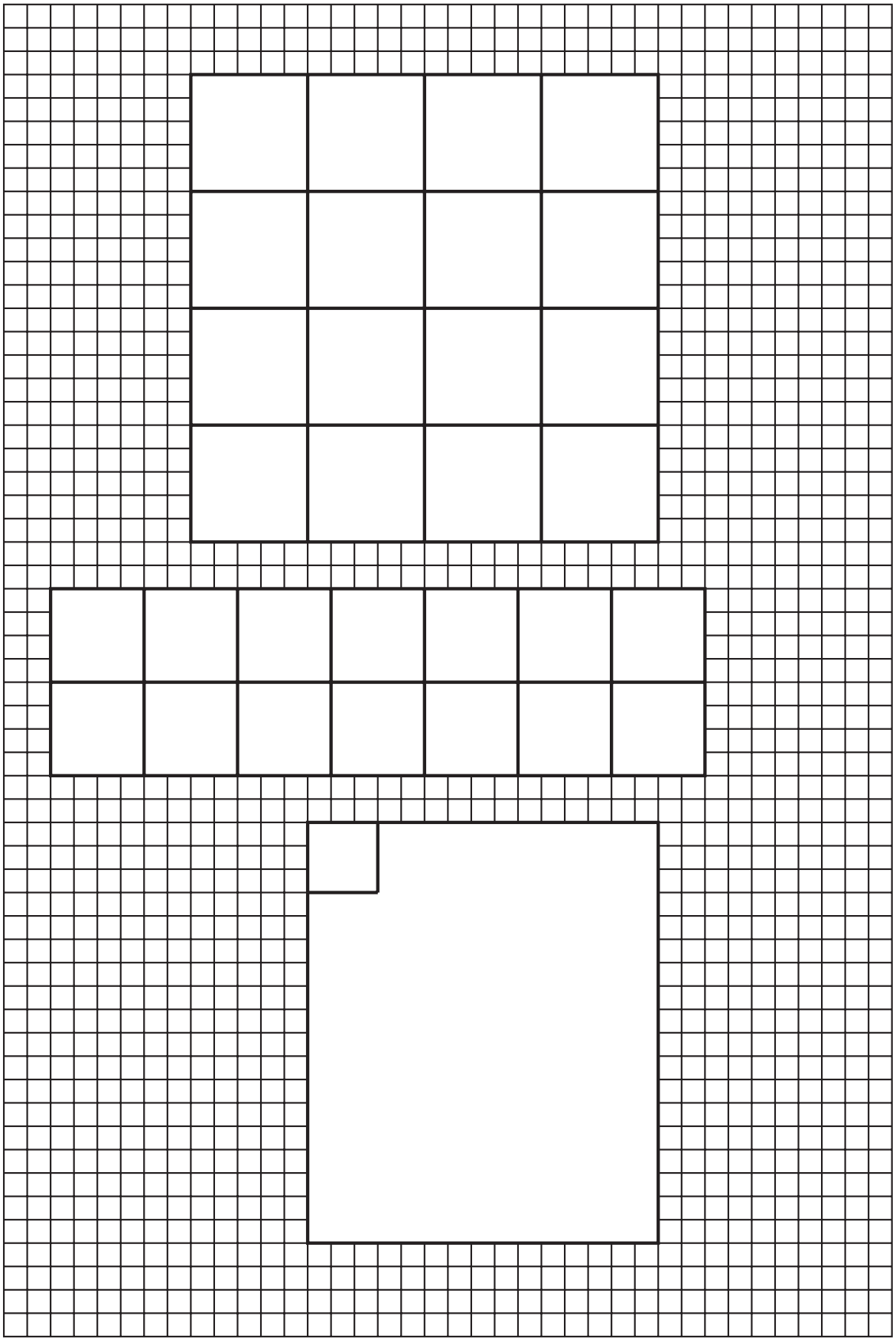
Students work out which rectangle has the largest area, without drawing grid lines.



Worksheet

Worksheet for 3.2: Hidden squares

How many small squares are hidden by the rectangles without counting every one of the small squares?



Worksheet

Teaching measurement: Volume and capacity

Volume and capacity

Volume is an extremely complex concept because of its different forms and the different ways of measuring it. The complexity of volume concepts means that many students are unlikely to develop conservation of volume until they have had experiences of measuring volume in a range of contexts, and with both types of units, liquid and cubic.

One aspect of volume is capacity (the term usually used for liquids measured in kilolitres, litres and millilitres) and means the amount a container can hold. The capacity of a jug might be measured in cups, millilitres or litres. The capacity of a large container, like a swimming pool could be measured in kilolitres or could be easily calculated in cubic metres from the internal dimensions of the pool if it is rectangular and its depth is consistent. Liquid measure may be judged on the basis of length. For example, if students add cups of water to a cylindrical container and predict how much the water level will rise each time, they usually base their judgments on a change in height. Capacity is similar to mass because the spatial structure of liquid measurement units is not important. For this reason, measuring capacity is easier than measuring volume where the spatial organisation of three-dimensional units must be considered.

A second aspect of volume is the volume of a model made with blocks. This aspect enables comparisons to be made easily—which model has more or fewer blocks—by counting the blocks. However, students may focus on counting and not associate this activity with volume. Nevertheless, it provides practice with stacking and layering, and may assist students to consider that not all blocks are visible when a model is built.

A third aspect of volume is the capacity (interior volume) of containers, when measured in cubic centimetres. The cubic units make this aspect of volume measurement more difficult than liquid measure. Students have to first learn to pack rectangular containers systematically. For example, students could measure and compare the volumes of their lunchboxes using informal cubic units, such as blocks. Packing activities develop knowledge of how the units are packed. This structure provides the basis for later development of the formula for the volume of a rectangular prism. In upper primary, students learn that one cubic centimetre is equal to one millilitre so they can see that both methods, packing or filling, give the same volume.

A fourth aspect of volume is exterior volume, or the amount of space a container takes up. Cosmetics containers are a good example. The amount the container holds (its interior volume) is often far less than the volume of the container (its exterior volume). Exterior volume of an object or irregularly shaped container is quite difficult to determine and would usually be found by displacement.

The fifth aspect of volume is that of displacement. This concept is difficult because it is not obvious that the amount of water displaced by an object is the volume of the object. However, experiences with displacement will assist students to develop this idea over time, especially if they measure identical objects so that the change in level, or the amount of water displaced is the same each time.

Working out the volume based on the number of blocks in a row or column of blocks and the number of rows in a layer depends on systematic packing. Knowledge of the structure of rows and columns in two dimensions, learnt as part of the area concept, will assist students to learn about the structure of three-dimensional packing. They might also use blocks connected together to make a "stick" to be equivalent to a row. The "sticks" can be used to form layers. Each "stick" would be a composite unit and so can be linked to repeated addition and multiplication. Students may be surprised at the large number of blocks in a container and they may need to check the number of layers and the number of blocks in each layer.

Experience with packing units, then drawing diagrams of these concrete situations, will help students perceive connections between textbook representations and practical situations. In the activities involving volume, rectangular containers are used so that students develop an understanding of the structure of the unit packing, in volume. When focusing on the unit structure, fractional units should be avoided if possible, as they may obscure the structural relationships.

Tasks can be made more challenging for students by changing the way that information is recorded or by asking them to estimate before measuring. For example, asking students to draw a three-dimensional array of cubes increases the difficulty of the task of packing a clear plastic container with cubes. Drawings may assist the teacher to see if students have grasped the structure of packing volume units.

Such experiences prepare students to learn at a later stage, that the three dimensions, length, breadth and height, of a rectangular container can be measured, and then multiplied together to find its volume. This strategy is a sophisticated one, because students have to perceive the relation between side lengths of a rectangular prism and volume to understand the basis for the formula.

Level descriptions for volume and capacity

Level 1	Knowledge and strategies
L1.1 Identification of the attribute Make direct comparisons of volume and capacity	1. use the terms, *pack*, *pour*, *fill*, *full* and *empty*, *nearly full* 2. recognise when a container is full and not full 3. pour or pack material into a container
L1.2 Identification of the attribute Order two or more quantities by direct comparison	1. be familiar with the terms *volume* and *capacity* and use comparative language, e.g. *larger*, *smaller*, *biggest*, *smallest*, *the same as* 2. use compensation strategies, e.g. one container is higher, but the other is wider so the two might hold about the same 3. compare volume or capacity systematically and explain why volume or capacity fits into a particular ordering
Level 2	**Knowledge and strategies**
L2.1 Informal measurement Choose and use appropriate units for measuring volume and capacity	1. pack a box with blocks and count the blocks; structure the packing in layers 2. fill a container by pouring and count the number of units used 3. state or record the number and type of units used to measure volume and capacity 4. suggest appropriate units and explain why one is better than another
L2.2 Informal measurement Compare and order volumes and capacities by filling or packing with identical units	1. compare capacities or volumes by filling or packing with identical units 2. know that the greater capacity or volume has more units 3. estimate the number of units and explain the estimation strategy 4. explain that the volume of fluid does not change when poured into containers of different sizes or shapes (conservation)

Level descriptions for volume and capacity

Level 3	Knowledge and strategies
L3.1 Structure of repeated units Use one unit or composite unit to work out how many will be needed altogether when making indirect comparisons	1. estimate then calculate the capacity of a container based on one unit (cups and 100 mL scoops) 2. explain the structure of cubic units packed into rectangular containers or packed to make a rectangular prism, e.g. three layers of six blocks 3. explain that the volume of material does not change when units are rearranged (conservation) 4. use change in level to compare the volume of objects (introduction to displacement)
L3.2 Structure of repeated units Explain the relationship between unit size and the number of units required to fill or pack a container	1. explain the relationship between unit size and number of units 2. express the same volume or capacity in different-sized units 3. know that measurement techniques must be consistent and precise

VOLUME & CAPACITY

Index of volume and capacity lesson ideas

Volume and capacity 1.1 lesson ideas

Identification of the attribute: make direct comparisons of volume and capacity

Knowledge and strategies

1. use the terms, *pack*, *pour*, *fill*, *full* and *empty*, *nearly full*
2. recognise when a container is full and not full
3. pour or pack material into a container

Fill it up

Students choose containers and materials for packing and pouring, so that each container is full, (filled to the brim). Suggestions for materials: sand, water, play dough.

Experiment with packing small containers or moulds with sand or play dough and then tipping out. Compare the moulded form with the original container.

Outcomes	Materials	Knowledge and strategies
MES1.3 WMES1.2	small containers and moulds, damp sand, modelling clay, water.	1. use the terms, *pack*, *pour*, *fill*, *full* and *empty*, *nearly full* 2. recognise when a container is full and not full 3. pour or pack material into a container

Gross comparisons

Find containers that would have a bigger or smaller capacity than a given container. Use terms such as *holds more*, *holds less*, *holds about the same*. The teacher models pouring and filling, emphasising the use of one kind of material. The students should predict which container or box will hold more or less than the other.

Outcomes	Materials	Knowledge and strategies
MES1.3 WMES1.2	a variety of containers and filling materials, (water, sand, marbles, rice, beans, packing foam, tennis balls, blocks)	1. use the terms, *pack*, *pour*, *fill*, *full* and *empty*, *nearly full* 2. recognise when a container is full and not full 3. pour or pack material into a container

Find a partner (see lesson plan)

Each student in the group is given a different container and must find someone with a container that will hold about the same amount of water or sand as their container. Check by pouring from one to the other.

Outcomes	Materials	Knowledge and strategies
MES1.3 WMES1.2	a variety of containers, jugs or buckets of water or sand	1. use the terms, *pack*, *pour*, *fill*, *full* and *empty*, *nearly full* 2. recognise when a container is full and not full 3. pour or pack material into a container

Packing them in

Provide students with a container and blocks. Students pack the blocks to determine who can fit the most blocks in the container.

Discuss the results and how the blocks can be packed without gaps.

Outcomes	Materials	Knowledge and strategies
MES1.3 WMES1.2 WMES1.3	rectangular or square containers, blocks for packing	1. use the terms, *pack*, *pour*, *fill*, *full* and *empty*, *nearly full* 2. recognise when a container is full and not full 3. pour or pack material into a container

What will it hold?

Students discuss which containers will hold different materials, e.g. a sieve will hold marbles but not sand or water and a round container is difficult to pack with blocks. Draw the results.

Outcomes	Materials	Knowledge and strategies
MES1.3 WMES1.3	a variety of containers (sieve, colander, boxes, cylinders) and units (water, sand, marbles, rice, beans, packing foam, blocks) for measuring, pencils and paper	1. use the terms, *pack*, *pour*, *fill*, *full* and *empty*, *nearly full* 2. recognise when a container is full and not full 3. pour or pack material into a container

Volume and capacity 1.1 lesson plan

Identification of the attribute: make direct comparisons of volume and capacity

Find a partner

Each student in the group is given a different container and must find someone with a container that will hold about the same amount of water or sand as their container. Check by pouring from one to the other.

Students should:

1. use the terms, *pack*, *pour*, *fill*, *full* and *empty*, *nearly full*
2. recognise when a container is full and not full
3. pour or pack material into a container

Grouping:

Step 1: whole-class discussion
Step 2: small group activity
Step 3: whole class reporting back

Outcomes

MES1.3 Compares the capacities of containers and the volumes of objects or substances using direct comparison.

WMES1.2 Uses objects, actions, imagery, technology and/or trial and error to explore mathematical problems.

Materials:

a variety of containers, jugs or buckets of water

Activity

Step 1

- Introduce the activity as the measurement of capacity, or how much a container will hold. Emphasise that the students are measuring.
- Explain that each student will choose a container and then work with a friend to find if one of the containers holds more, or if they have about the same capacity.
- Teacher or student demonstrate how to fill the container, and pour from one container to another.

Questioning

Can you show us what a full cup of water looks like? How do you know it is full?

Can you guess which of these two cups will hold more water? How could we find out?

What does it mean if all of the water in my cup won't fit into my friend's cup?

How will we know if the cups hold about the same amount?

Step 2

- Students work in groups of 4–6 to find a partner whose container has approximately the same capacity.
- Record results.

Check that students:

- pour accurately
- know when their container is full
- discuss full, not full, holds more with their partner.

Step 3

- Whole class watches while pairs demonstrate their matching containers.

Discussion

Did you find a partner whose cup has about the same capacity?

Can we say the cups match if a lot of water pours over the top?

What were we measuring?

Volume and capacity 1.2 lesson ideas

Identification of the attribute: order two or more quantities by direct comparison

Knowledge and strategies

be familiar with the terms *volume* and *capacity* and use comparative language, e.g. *larger*, *smaller*, *biggest*, *smallest*, *the same as*

use compensation strategies, e.g. one container is higher, but the other is wider so the two might hold about the same

compare volume or capacity systematically and explain why volume or capacity fits into a particular ordering

Who can hold the most? (see lesson plan)

Students work in small groups to find who can hold the most beans in two hands (cupped together). Students compare the volume of their handfuls by putting their beans into clear plastic cups and looking at the height of the beans in the cup. Some students may be able to count the beans.

Variation: Do the same activity with blocks and count them.

Outcomes

MES1.3

WMES1.2

Materials

lima beans, clear plastic cups, pencils and paper

Knowledge and strategies

1. be familiar with the terms *volume* and *capacity* and use comparative language, e.g. *larger*, *smaller*, *biggest*, *smallest*, *the same as*
2. use compensation strategies, e.g. one container is higher, but the other is wider so the two might hold about the same
3. compare volume or capacity systematically and explain why volume or capacity fits into a particular ordering

Let's compare

Find three containers that will hold more water than a given container and three that will hold less water.

Variations: Use objects instead of containers. Find objects that take up more space than … Find bottles that have the same capacity (give students a selection of bottles which have different shapes and bases).

Outcomes

MES1.3

WMES1.2

Materials

a range of containers and water, objects in the classroom, stacking dolls, boxes, etc.

Knowledge and strategies

1. be familiar with the terms *volume* and *capacity* and use comparative language, e.g. *larger*, *smaller*, *biggest*, *smallest*, *the same as*
2. use compensation strategies, e.g. one container is higher, but the other is wider so the two might hold about the same
3. compare volume or capacity systematically and explain why volume or capacity fits into a particular ordering

Towers

Build different towers and discuss which tower takes up more or less space (has greater, lesser volume).

Outcomes	Materials	Knowledge and strategies
MES1.3 WMES1.3	equal-size blocks	1. be familiar with the terms *volume* and *capacity* and use comparative language, e.g. *larger*, *smaller*, *biggest*, *smallest*, *the same as* 2. use compensation strategies, e.g. one container is higher, but the other is wider so the two might hold about the same 3. compare volume or capacity systematically and explain why volume or capacity fits into a particular ordering

Can it be?

Have two different-sized cans or plastic containers, one short and fat and the other one tall and skinny. Fill one container with sand or rice. Students then predict which container will hold the most or has the greatest capacity. Pour the contents from the first container into the other one. Record the results.

Outcomes	Materials	Knowledge and strategies
MES1.3 WMES1.2	two cans or plastic containers, rice or sand, pencils and paper	1. be familiar with the terms *volume* and *capacity* and use comparative language, e.g. *larger*, *smaller*, *biggest*, *smallest*, *the same as* 2. use compensation strategies, e.g. one container is higher, but the other is wider so the two might hold about the same

Find a container

Whole-class activity in which students are shown a selection of four or five containers and a packet of wrapped lollies. Students predict which container will be filled to the brim by the lollies. Students explain their predictions. The containers are trialled and discussed and students who predicted correctly receive a lolly.

Outcomes	Materials	Knowledge and strategies
MES1.3 WMES1.2	range of containers, wrapped lollies	1. be familiar with the terms *volume* and *capacity* and use comparative language, e.g. *larger*, *smaller*, *biggest*, *smallest*, *the same as* 2. use compensation strategies, e.g. one container is higher, but the other is wider so the two might hold about the same 3. compare volume or capacity systematically and explain why volume or capacity fits into a particular ordering

Volume and capacity 1.2 lesson plan

Identification of the attribute: order two or more quantities by direct comparison

Who can hold the most?

Students work in small groups to find who can hold the most beans in one hand. Students compare the volume of their handfuls by putting their beans into clear plastic cups and looking at the height of the beans in the cup. Some students may be able to count the beans. Variation: Do the same activity with blocks and count them.

Students should:

1. be familiar with the terms *volume* and *capacity* and use comparative language, e.g. *larger*, *smaller*, *largest*, *smallest*, *the same as*
2. use compensation strategies, e.g. one container is higher, but the other is wider so the two might hold about the same
3. compare volume or capacity systematically and explain why volume or capacity fits into a particular ordering

Grouping:

Step 1: whole-class discussion

Step 2: small groups to compare handfuls

Step 3: groups report back to the whole class.

Materials:

lima beans or similar material, clear plastic cups, pencils and paper

Outcomes

MES1.3 Compares the capacities of containers and the volumes of objects or substances using direct comparison

WMES1.2 Uses objects, actions, imagery, technology and/or trial and error to explore mathematical problems.

Activity

Step 1

- Introduce the measurement lesson.
- Whole-class discussion on comparing the volume of material which students can hold in one hand, or finding who can hold the most in their hand (capacity).

Questioning

What could we measure about a hand? (length, area, capacity)

How could we work out who can hold the most in one hand?

What materials might be good to use?

How could we make an order from largest hand to smallest hand in our groups?

What are the words we will need to use?

(bigger than, smaller than, more, less, as much as)

Step 2

A possible routine in groups of 4–6

- Students estimate how many beans or other materials they will be able to hold. Record on a small label with name.
- Students dip into a container of beans, fill their hand and then put their beans or materials into clear plastic cups to compare.
- Students add their name label, so the cups can be identified.
- Order the cups.
- Confirm order by counting the beans.

Check that students:

- estimate first
- take a handful of material
- can compare the volume of material in cups
- understand that the student who holds the most beans has the largest hand capacity.

Step 3

- Class discussion of answering the question: "Who can hold the most?"

Discussion

Did your group order well?

Can you put the counted numbers into order, as well?

Are the beans good for measuring capacity? Why? Why not?

What else could we use for measuring the capacity of hands?

Volume and capacity 2.1 lesson ideas

Informal measurement: choose and use appropriate units for measuring volume and capacity

Knowledge and strategies

1. pack a box with blocks and count the blocks; structure the packing in layers
2. fill a container by pouring and count the number of units used
3. state or record the number and type of units used to measure volume and capacity
4. suggest appropriate units and explain why one is better than another

What's the capacity?

Students choose a unit (cup, jug, egg cup, yoghurt container), to measure the capacity of a bucket to be filled with water or sand. Count and record the number of units used. Comment on the suitability of the unit.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.2	empty bucket, water or sand, variety of containers to use as units	2. fill a container by pouring and count the number of units used 3. state or record the number and type of units used to measure volume and capacity 4. suggest appropriate units and explain why one is better than another

Fill the box (see lesson plan)

Students pack boxes with blocks. Then they count the blocks and discuss, draw and write about the structure of their packing. Emphasise layers, rows and columns (boxes may have been packed in horizontal or vertical layers).

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.3	boxes, plastic containers, blocks, paper and pencil	1. pack a box with blocks and count the blocks; structure the packing in layers 3. state or record the number and type of units used to measure volume and capacity 4. suggest appropriate units and explain why one is better than another

How will I pack?

Pack clear perspex containers with different materials, e.g. sand, rice, marbles, blocks, and discuss which is better to fill the containers without leaving gaps.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.5	sand, rice, marbles, blocks, tennis balls	1. pack a box with blocks and count the blocks; structure the packing in layers 2. fill a container by pouring and count the number of units used 4. suggest appropriate units and explain why one is better than another

How could I measure?

Students suggest different materials that could be used to measure different containers, e.g. sand, water for cylindrical containers, blocks for rectangular boxes. Record what happened when different materials were used.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.2	sand, rice, marbles, blocks, beans, containers, pencils and paper	1. pack a box with blocks and count the blocks; structure the packing in layers 2. fill a container by pouring and count the number of units used 3. state or record the number and type of units used to measure volume and capacity 4. suggest appropriate units and explain why one is better than another

What will happen?

Students add half a cup of sand or half a cup of water to half a cup of marbles. Explain what happens and why.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.4	sand, rice, marbles, blocks, beans	2. fill a container by pouring and count the number of units used 4. suggest appropriate units and explain why one is better than another

Volume and capacity 2.1 lesson plan

Informal measurement: choose and use appropriate units for measuring volume and capacity

Fill the box

Students pack boxes with blocks. Then they count the blocks and discuss, draw and write about the structure of their packing. Emphasise layers, rows and columns (boxes may have been packed in horizontal or vertical layers).

Students should:

1. pack a box with blocks and count the blocks; structure the packing in layers
3. state or record the number and type of units used to measure volume and capacity
4. suggest appropriate units and explain why one is better than another

Grouping:

Step 1: whole-class discussion
Step 2: work in groups of 2 or 3
Step 3: report back to the class.

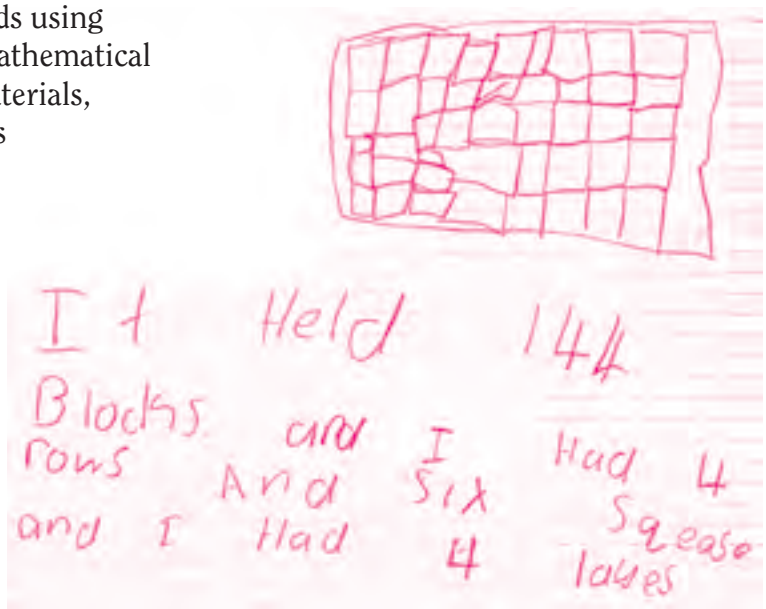
Outcomes

MS1.3 Estimates, measures, compares and records volumes and capacities using informal units

WMS1.3 Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols

Materials:

boxes, plastic containers, blocks, paper and pencils for recording



Activity

Step 1

- Introduce the activity as measuring the *capacity* of containers using blocks.
- Discuss the structure of packing. Teacher model the use of "rows", "columns" and "layers". Remind students that layer patterns will look like area grids.
- Whole class discuss and count the blocks in one layer. Discuss how to find the total number of blocks.

Questioning

If I want to measure the capacity of this container by using the blocks, how should I put them in?

What pattern are the blocks in? Could we fit any more in for this layer?

What does the pattern of this layer look like?

How many blocks have we got now? How many more do you think we will need? Is there a quick way of counting the blocks we use?

Step 2

- Students work with a partner or small group to choose units, then pack these into their box.
- Students draw their packed box and write the total number of blocks used to measure the capacity. Students record how they packed the box.

Check that students:

- pack the blocks methodically
- understand that there should be no gaps
- understand how to count: multiplication, skip counting or stress counting.

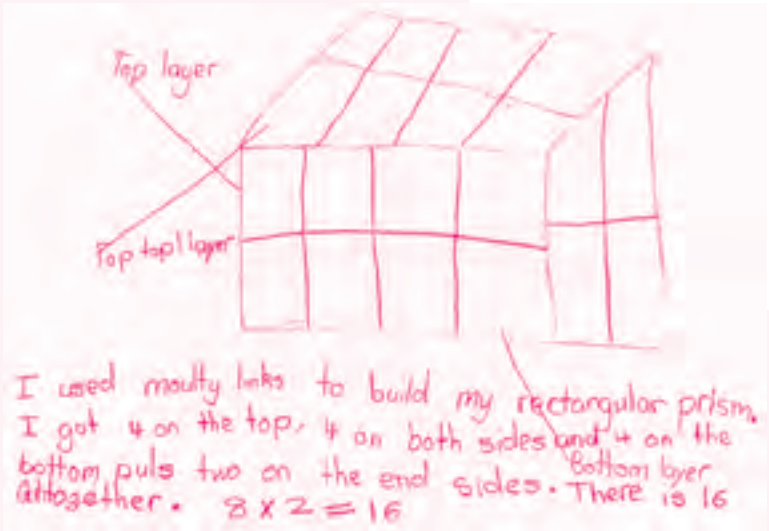
Step 3

- Discuss the suitability of the blocks, compared with lima beans or marbles.

Discussion

Were the blocks good for measuring capacity of the boxes? Why?

How did you count the blocks? Was there an easy way to draw the pattern of the layers?



Volume and capacity 2.2 lesson ideas

Informal measurement: compare and order volumes and capacities by filling or packing with identical units

Knowledge and strategies

1. compare capacities or volumes by filling or packing with identical units
2. know that the greater capacity or volume has more units
3. estimate the number of units and explain the estimation strategy
4. explain that the volume of fluid does not change when poured into containers of different sizes or shapes (conservation)

Compare by pouring

Order the capacities of containers (drink bottles, lunch boxes) by estimating first, then filling with water, sand, etc. Students need to use the same materials and units of measure for all containers.

Record the estimates and actual measure in words and drawings.

Outcomes

MS1.3
WMS1.2

Materials

a variety of packing materials and containers, pencils and paper

Knowledge and strategies

1. compare capacities or volumes by filling or packing with identical units
2. know that the greater capacity or volume has more units
3. estimate the number of units and explain the estimation strategy

Cylinders! (see lesson plan)

Cut a sheet of cardboard into two equal parts. Make one tall and one short cylinder. Ensure there is tape across the bottom of the cylinders. Estimate, then measure the capacity of the two cylinders. Discuss the results.

Outcomes

MS1.3
WMS1.2
WMS1.3

Materials

cardboard sheets, scissors, tape, lima beans or similar packing material or sand, pencils and paper

Knowledge and strategies

1. compare capacities or volumes by filling or packing with identical units
2. know that the greater capacity or volume has more units
3. estimate the number of units and explain the estimation strategy
4. explain that the volume of fluid does not change when poured into containers of different sizes or shapes (conservation)

Compare by packing

Order rectangular containers by estimating, then packing with blocks. Record the number of blocks used and a diagram of the packing. Students should draw the array as accurately as possible.

Outcomes

MS1.3
WMS1.3

Materials

variety of rectangular containers, blocks, pencils and paper

Knowledge and strategies

1. compare capacities or volumes by filling or packing with identical units
2. know that the greater capacity or volume has more units
3. estimate the number of units and explain the estimation strategy

Don't be tricked

Students are given four containers and a specific number of blocks. Students estimate which container would be filled by the blocks and then check by measuring. Record results.

Extension

Use a different set of containers to predict, then measure, the containers which will be half-filled by the blocks.

Outcomes

MS1.3
WMS1.4

Materials

a variety of different-sized containers, blocks, pencils and paper

Knowledge and strategies

1. compare capacities or volumes by filling or packing with identical units
2. know that the greater capacity or volume has more units
3. estimate the number of units and explain the estimation strategy

Snakes

Students predict how many snakes they can make from a ball of modelling clay. Compare the number of snakes made by each student in a group. Ask students whether they think that everyone had the same amount of modelling clay to work with. Students roll their snakes back into a ball and compare.

Outcomes

MS1.3
WMES1.1
WMES1.2

Materials

identical balls of modelling clay

Knowledge and strategies

3. estimate the number of units and explain the estimation strategy
4. explain that the volume of fluid does not change when poured into containers of different sizes or shapes (conservation)

Volume and capacity 2.2 lesson plan

Informal measurement: compare and order volumes and capacities by filling or packing with identical units.

Cylinders

Cut a sheet of cardboard into two equal parts. Make one tall and one short cylinder. Ensure there is tape across the bottom of the cylinders. Estimate, then measure the capacity of the two cylinders. Discuss the results.

Students should:

1. compare capacities or volumes by filling or packing with identical units
2. know that the greater capacity or volume has more units
3. estimate the number of units and explain the estimation strategy
4. explain that the volume of fluid does not change when poured into containers of different sizes or shapes (conservation)

Grouping:

Step 1: whole-class discussion and demonstration

Step 2: work in pairs

Step 3: report back to whole class.

Materials:

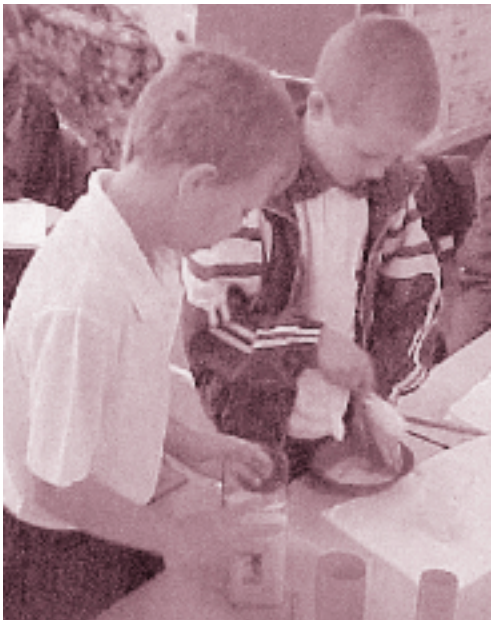
cardboard sheets, scissors, tape, lima beans or similar packing material or sand, pencils and paper

Outcomes

MS1.3 Estimates, measures, compares and records volumes and capacities using informal units.

WMS1.1 Asks questions that could be explored using mathematics in relation to Stage 1 content.

WMS1.2 Uses objects, diagrams, imagery and technology to explore mathematical problems.



Activity

Step 1

- Introduce the activity as measuring the capacity of cylinders and explain the term estimating and what this means.
- Discuss what a cylinder is and how it can be made from cardboard.
- Demonstrate how to make a cylinder.
- Discuss suitable measuring material for the cylinder (no gaps).

Step 2

- Students work with a partner to make two different cylinders from their cardboard strip, one short and one tall.
- Estimate which one will hold the most and record their estimates.
- Choose appropriate materials for measuring the capacity of the container.
- Students fill the cylinders, counting the number of units used.
- Record the results by drawing the cylinders and explaining the results.
- Compare with their estimates.

Step 3

- Discuss which container held the most or least

Questioning

What is a cylinder?

Where can we find cylinders in the classroom.

What words describe the appearance of different cylinders?

How can I make a cylinder from cardboard?

What does estimation mean?

How can we check our estimates?

Check that students:

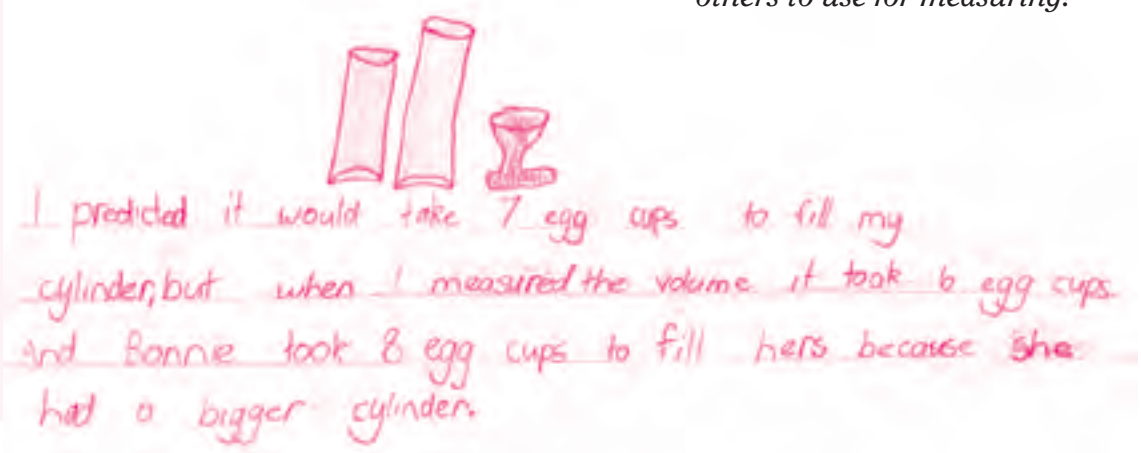
- tape the cylinder firmly together
- tape the end of the cylinder so the contents will not go straight through
- record their estimates before measuring
- choose suitable materials to measure
- record the results.

Discussion

Which container held the most? Why?

Were the estimates close?

Were some materials better than others to use for measuring?



Volume and capacity 3.1 lesson ideas

Structure of repeated units: use one unit or composite unit to work out how many will be needed altogether when making indirect comparisons

Knowledge and strategies

1. estimate then calculate the capacity of a container based on one unit (cups and 100 mL scoops)
2. explain the structure of cubic units packed into rectangular containers or packed to make a rectangular prism (e.g. three layers of six blocks)
3. explain that the volume of material does not change when units are rearranged (conservation)
4. use change in level to compare the volume of objects (introduction to displacement)

Let's estimate

Estimate how many units (100 mL scoops or cups), it will take to fill a container having marked on its side how much one cup is. Measure to check and record.
Variation: Use a variety of containers, e.g. tall, short, square, round.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.4	container marked to show the water level of one cup, a cup, water, pencils and paper	1. estimate then calculate the capacity of a container based on one unit (cups and 100 mL scoops)

How high?

Students start with an empty container and a cup measure or 100 mL scoop. Estimate the height of the water level from one cup, and mark the container. Add a cup of water and mark the level. Compare the estimate and true level, and repeat estimating and measuring until the container is full. Use two different colours to mark the estimate and the measure.
Variation: Repeat with a variety of containers, e.g. short, wide, tall, square.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.2	cup or 100 mL scoop, large empty container, pens, water, pencils and paper	1. estimate then calculate the capacity of a container based on one unit (cups and 100 mL scoops)

In they go

Students have different materials, e.g. golf balls, marbles. They estimate how many of their units can be added to a container of water so it will be just filled without overflowing.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.2	large containers, water, marbles, golf balls (items which have the same volume and will sink)	4. use change in level to compare the volume of objects (introduction to displacement)

Displacement (see lesson plan)

Students see which rock has the greatest volume by immersing rocks one-by-one in a container of water and measuring the amount of water displaced. Different students could measure different rocks. Encourage students to estimate and mark the new height of water. This activity could also be done with other objects, providing they sink.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.2	a variety of rocks of different sizes, containers, water, pencils and paper	4. use change in level to compare the volume of objects (introduction to displacement)

How many will fit?

Students are given a small number of blocks and asked to work out how many would be needed to fill a rectangular container such as their lunchbox. Students calculate the total by counting the number of blocks in each layer and the number of layers, e.g. 2 x 3 x 3; 1 x 3 x 3.

Students build a different model using the same number of blocks.

Outcomes	Materials	Knowledge and strategies
MS1.3 NS1.3 WMS1.2	clear rectangular containers and cubes, pencils and paper	2. explain the structure of cubic units packed into rectangular containers or packed to make a rectangular prism (e.g. three layers of six blocks) 3. explain that the volume of material does not change when units are rearranged (conservation)

How many cubes?

Students make simple rectangular, solid constructions using a given number of cubes. Other students have to work out, without touching, how many cubes were used. Discuss the strategies students used to calculate the volume of the construction.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.3 WMS1.4	cubes	2. explain the structure of cubic units packed into rectangular containers or packed to make a rectangular prism (e.g. three layers of six blocks)

Volume and capacity 3.1 lesson plan

Structure of repeated units: use one unit or composite unit to work out how many will be needed altogether when making indirect comparisons

Displacement

Students see which rock has the greatest volume by immersing them one-by-one in a container of water and measuring the amount of water displaced. Different students could measure different rocks. Encourage students to estimate and mark the new height of water.

Students should:

4. Use change in level to compare the volume of objects (introduction to displacement)

Grouping:

Step 1: whole-class discussion and demonstration

Step 2: work in small groups

Step 3: report back to whole class.

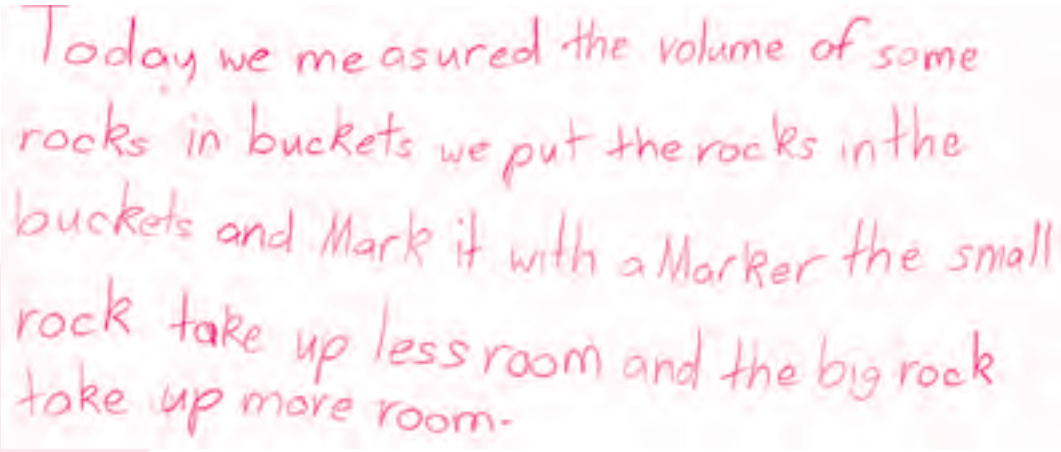
Outcomes

MS1.3 Estimates, measures, compares and records volumes and capacities using informal unit.

WMS1.2 Uses objects, actions, imagery and technology to explore mathematical problems.

Materials:

a variety of rocks of different sizes, containers, soluble marking pens, water



Activity

Step 1

- Introduce the activity as measuring the *volume* of objects by displacement. Relate to what happens when someone gets into a bath of water.
- Teacher or student demonstrate how to mark the level of water and then drop a rock into the container.
- Students estimate the volume of a second rock, perhaps a short fat rock that had a larger volume than a tall rock.

Questioning

What will happen when I gently drop this rock into the jug of water? Why?

About how much do you think the water level will rise? Why?

How could we measure how much the water level rises?

What is your estimate for this rock? Will the water level be higher or lower? What does it mean if the water level is higher?

Step 2

- Students work in a small group to estimate, then measure the volume of rocks or other objects using displacement. Measure the increase in height of the water level, or the volume of water which pours out of the vessel.
- Record the process and the results.

Check that students:

- are able to explain their estimates
- understand that the amount of water displaced is equal to the volume of the submerged object
- understand that the larger the volume of the rock the more water it displaces.

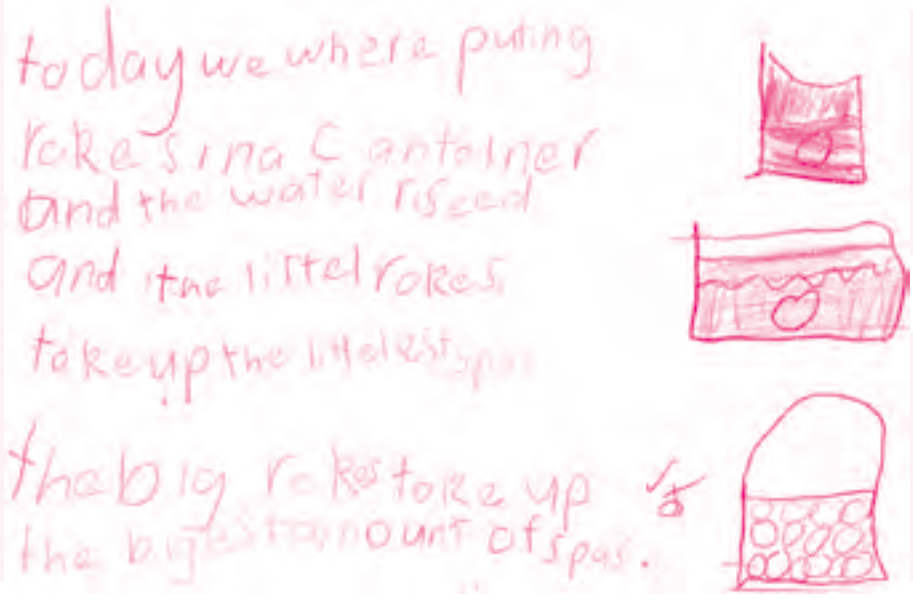
Step 3

- Discuss the use of displacement for measuring *volume*.

Discussion

Could you find the volume of a tennis ball or an apple by displacement?

What else could we measure, in the classroom?



Volume and capacity 3.2 lesson ideas

Structure of repeated units: explain the relationship between unit size and the number of units required to fill or pack a container

Knowledge and strategies

1. explain the relationship between unit size and number of units
2. express the same volume or capacity in different-sized units
3. know that measurement techniques must be consistent and precise

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Planning a party (see lesson plan)

Students work out how many different-sized cups of drink could be obtained from one bottle of soft drink. Students then work out how many bottles of soft drink they would need to buy for a class or family party.

Outcomes

MS1.3
WMS1.2

Materials

soft drink bottles, paper or plastic cups, pencils and paper

Knowledge and strategies

1. explain the relationship between unit size and number of units
2. express the same volume or capacity in different-sized units
3. know that measurement techniques must be consistent and precise

Work it out

Students estimate, measure and record how many egg cupfuls, cupfuls or mugfuls would fit into a container once they have been told the capacity in terms of one of these units.

Outcomes

MS1.3
WMS1.2

Materials

eggcups, cups, mugs, large containers, pencils, paper

Knowledge and strategies

1. explain the relationship between unit size and number of units
2. express the same volume or capacity in different-sized units
3. know that measurement techniques must be consistent and precise

Count and compare

Students select a rectangular box and interlocking blocks suitable for measuring the volume by packing. Students find the volume by using single blocks as units, then predict how many units will be required when blocks are joined in twos or fours. Record estimate before investigating.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.3	rectangular containers, interlocking blocks, pencils, paper	1. explain the relationship between unit size and number of units 2. express the same volume or capacity in different-sized units 3. know that measurement techniques must be consistent and precise

How many marbles?

Compare the volume of different units by using displacement. Half-fill two identical containers with water and mark the water levels. Put a cricket ball in one container and then mark the displacement level. Estimate how many marbles would have the same volume as the ball and then measure by adding the marbles until the water levels are equal. (Estimation can be revised after five marbles have been added). Estimate and check how many marbles would displace the same volume of water as two cricket balls.

Note: Marbles need to be the same size. Other small objects could be used, e.g. golf balls, bolts.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.4	containers, water, cricket balls, marbles or other small heavy objects, pencils and paper	1. explain the relationship between unit size and number of units 2. express the same volume or capacity in different-sized units 3. know that measurement techniques must be consistent and precise

Making a measuring container

Students mark gradations on a large container for two different units (e.g. cupfuls and mugfuls).

Estimate, then measure the capacities of containers in terms of the two different units. Explain the difference in measurements.

Outcomes	Materials	Knowledge and strategies
MS1.3 WMS1.3	containers and different measuring units (cups, mugs, egg cups, tablespoons), pencils and paper	1. explain the relationship between unit size and number of units 2. express the same volume or capacity in different-sized units 3. know that measurement techniques must be consistent and precise

Volume and capacity 3.2 lesson plan

Structure of repeated units: explain the relationship between unit size and the number of units required to fill or pack a container

Planning a party

Students work out how many different-sized cups of drink could be obtained from one bottle of soft drink. Students then work out how many bottles of soft drink they would need to buy for a class or family party.

Students should:

1. explain the relationship between unit size and number of units
2. express the same volume or capacity in different-sized units
3. know that measurement techniques must be consistent and precise

Grouping:

Step 1: whole-class discussion

Step 2: work in small groups

Step 3: report back to whole class.

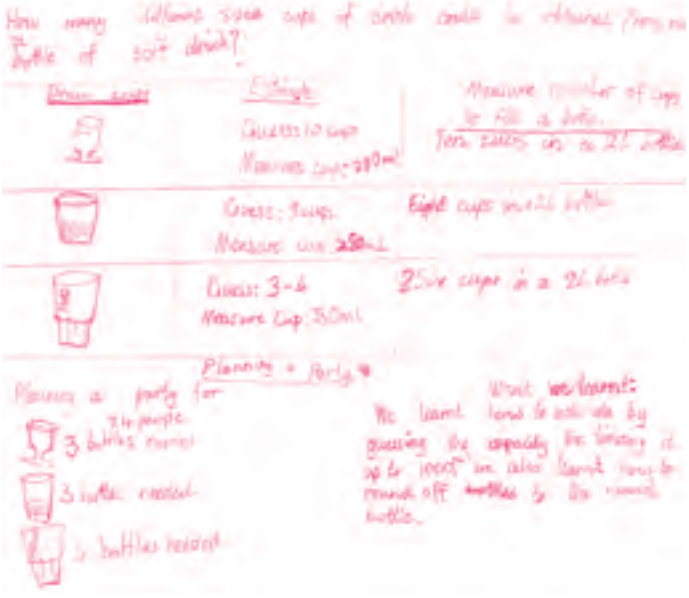
Outcomes

MS1.3 Estimates, measures, compares and records volumes and capacities using informal units.

WMS1.2 Uses objects, actions, imagery and technology to explore mathematical problems.

Materials:

soft drink bottles, paper or plastic cups in at least two different sizes for each group, water or sand, pencils and paper



Activity

Step 1

- Introduce the lesson as the comparison of the *capacity* of different-sized cups that could be used for a class party.
- Select a cup and ask the students to estimate how many cupfuls can be poured from a bottle of drink. Commence pouring and allow students to revise their estimate after 2 or 3 cupfuls.
- Discuss how the capacity of different-sized cups will affect the number of cupfuls in the bottle. Select a larger or smaller cup and compare the capacity with the first cup. Use the comparison to estimate how many of the new cups can be poured from the bottle. Check by pouring.
- Explain that the students will calculate how many bottles of soft drink would be needed for a class party, according to the cups selected.

Questioning

How could we compare the capacity of these different-sized cups?

How many cupfuls do you think we will pour from the bottle?

If I use a different-sized cup, how many cupfuls will we get from the same bottle of drink?

How will the size of the cup affect the number of bottles of drink that we need to buy?

How could you work out the number of bottles of drink required for a class party?

Step 2

- Students in small groups select one cup to estimate, then measure how many cupfuls can be poured from the bottle.
- Students calculate and record the number of cupfuls for a larger or smaller cup, by comparing the capacity with the first measure. Check by pouring.
- Students estimate and calculate how many bottles of soft drink would be needed for a class party, for at least two cup sizes.

Check that students:

- measure and count accurately
- understand that the size of the cup will determine the number of cupfuls poured from each bottle
- estimate the number of bottles of drink needed for the whole class
- record their results.

Step 3

- Students report back to the whole class.
- Discuss difficulties that may have occurred with partly-filled bottles.

Discussion

What decisions did you have to make?

How could you be sure of your measurement?

What is the best-sized cup for a class party? Why?

What else would you need to calculate for a class party?

VOLUME & CAPACITY

Teaching measurement: Mass

Mass

Mass is defined as the amount of matter in an object but, like time and temperature, it cannot be seen. Students may confuse mass and volume because objects with a larger volume will often have more mass than those with a smaller volume. However, if two contrasting materials are compared, for example, foam packaging and iron, students will quickly realise that the larger volume does not necessarily have the larger mass.

Mass may also be confused with weight. Students may have some awareness of the difference between these two concepts now that space travel has been widely reported and astronauts have been shown to weigh less on the moon because the moon's gravitational field is not as strong as that of earth. Weight is a force that is affected by gravity and so as gravity changes, the weight of an object will change. So scientifically, it is incorrect to say an object weighs one kilogram because weight is measured in units of force (Newtons, named after Sir Isaac Newton who formulated the law of gravity). The correct expression is that an object has a mass of one kilogram. Use of precise language by teachers will assist students to distinguish between these two concepts when they encounter them in secondary school.

Young students begin to directly compare masses by hefting them. Once they have had experience comparing two masses by hefting, students will have a sense of what balance means, especially if they try to heft one mass that is much heavier than the other so that their arms feel "pushed down" on one side.

The measurement framework for mass is slightly different from length, area and volume. Units of mass are not spatially organised and students must learn to use an equal-arm balance to measure mass. Level 1 has been extended to include the use of an equal arm balance in 1.3. When using informal units, students add units until they balance the mass being measured.

The only concept at Level 3 is that of the relationship between the size of the unit and the number of units. For example, if four-block units are used to balance an object, then four times as many will be needed if one-block units are substituted.

Level descriptions for mass

Level 1	Knowledge and strategies
L1.1 Identification of the attribute Make direct comparisons of mass	1. use mass vocabulary such as *light*, *heavy*, *hard to push* or *lift*, 2. identify materials that are light and heavy 3. identify objects that have about the same mass 4. heft or push objects and decide which is heavier or lighter
L1.2 Identification of the attribute Compare and order objects by hefting	1. use comparative language: *heavier than*, *lighter than*, *heaviest*, *lightest* 2. predict which object would be heavier than, lighter than, about the same and explain reasoning. 3. compare masses systematically and explain why a mass fits into a particular ordering
L1.3 Identification of the attribute Compare masses using an equal-arm balance	1. know that an identical object on each side will balance 2. know that the side that goes down has the greater mass 3. demonstrate how to use an equal-arm balance to compare three or more objects and report the results
Level 2	**Knowledge and strategies**
L2.1 Informal measurement Choose appropriate units and use them to measure a mass	1. find out how many identical units will balance a given mass 2. suggest appropriate units and explain why some units would be better than others 3. know that spatial structure is not important for mass and that round objects such as marbles are suitable for measuring mass
L2.2 Informal measurement Compare and order masses using identical units for each mass	1. choose identical units to measure each mass 2. explain that the heavier mass has more units 3. estimate the number of units and explain the estimation strategy 4. know that mass is conserved if rearranged
Level 3	**Knowledge and strategies**
L3.2 Relationship between units Explain the relationship between unit size and number of units required to balance a mass	1. explain the relationship between unit size and the number of units 2. express the same mass in terms of different-sized units 3. know that measurement techniques must be consistent and precise

MASS

Index of mass lesson ideas

Mass 1.1 lesson ideas

Identification of attribute: make direct comparisons of mass

Knowledge and strategies

1. use mass vocabulary such as *light*, *heavy*, *hard to push* or *lift*,
2. identify materials that are light and heavy
3. identify objects that have about the same mass
4. heft or push objects and decide which is heavier or lighter

Two groups

Sort heavy and light objects into two groups. Objects should be obviously light or obviously heavy, e.g. piece of string, paperclip, large stone, large bottle of liquid.

Students report back on why objects were placed into different groups.

Outcomes	Materials	Knowledge and strategies
MES1.4 WMES1.2	selected objects	1. use mass vocabulary such as *light*, *heavy*, *hard to push* or *lift*, 2. identify materials that are light and heavy 4. heft or push objects and decide which is heavier or lighter

Hefting

Heft masses and describe which is heavier and which is lighter (when the mass is not related to the size of the packaging, e.g. feather, tennis balls, books).

Outcomes	Materials	Knowledge and strategies
MES1.4 WMES1.3	various balls (tennis ball, ping pong ball, golf ball, football), large leaves or flowers, feathers, large and small books	1. use mass vocabulary such as *light*, *heavy*, *hard to push* or *lift*, 2. identify materials that are light and heavy 3. identify objects that have about the same mass 4. heft or push objects and decide which is heavier or lighter

Twin bags (see lesson plan)

Find a partner who has a bag with about the same mass as your bag. Students are each given a bag containing a mass. Teacher prepares the bags or invites students to fill the bags, given a choice of objects and materials.

Outcomes	Materials	Knowledge and strategies
MES1.4 WMES1.2	cloth bags or opaque plastic bags, materials for the bags	1. use mass vocabulary such as *light*, *heavy*, *hard to push* or *lift*, 3. identify objects that have about the same mass 4. heft or push objects and decide which is heavier or lighter

Blindfold

Students take turns to be blindfolded. Teacher or another student places an object or container in each hand of the blindfolded student. Objects should be obviously light or obviously heavy objects e.g. piece of string, paperclip, large stone, large bottle of liquid.

Students state which hand is holding the heavier object or container. Students watching make a visual estimate of which is the heavier object.

Outcomes	Materials	Knowledge and strategies
MES1.4 WMES1.2	selected objects	1. use mass vocabulary such as *light*, *heavy*, *hard to push* or *lift*, 2. identify materials that are light and heavy 3. identify objects that have about the same mass

Heavy bag, light bag

Take turns to decide if your bag is heavier, lighter or the same as the teacher's. (The teacher's bag should be lighter, heavier or about the same mass as students' bags.)

Outcomes	Materials	Knowledge and strategies
MES1.4 WMES1.3	students' own school bags or another bag and a bag packed by the teacher	1. use mass vocabulary such as *light*, *heavy*, *hard to push* or *lift*, 3. identify objects that have about the same mass 4. heft or push objects and decide which is heavier or lighter

Mass 1.1 lesson plan

Identification of attribute: make direct comparisons of mass

Twin bags

Students are each given a bag containing a mass. Students find a partner who has a bag with about the same mass as theirs.

Students should:

1. use mass vocabulary such as *light*, *heavy*, *hard to push* or *lift*,
3. identify objects that have about the same mass
4. heft or push objects and decide which is heavier or lighter.

Grouping:

Step 1: whole-class discussion

Step 2: work in groups of two or three

Step 3: report back to the class.

Outcomes

MES1.4 Compares the masses of two objects and describes mass using everyday language.

WMES1.2 Uses objects, actions, imagery, technology and/or trial and error to explore mathematical problems.

Materials:

cloth bags or opaque plastic bags, materials for the bags

MASS

Activity

Step 1

- Introduce the activity as measuring mass.
- Discuss where mass might be measured: shopping, sports teams, new babies.
- Teacher demonstrates how to heft two items. Describe the sensation and introduce vocabulary for comparing two masses. Invite students to model and talk about what they feel when they heft. Write mass vocabulary on the chalkboard.
- Introduce the task: Students each have a bag and work in small groups to find two bags which have similar mass. Discuss how to record the activity.

Step 2

Activity: Students take turns in their group to heft the bags.

Students record the activity by drawing themselves and the bags and labelling.

Step 3

Report back to the class about bags that were heavy, light, about the same mass.

Questioning

What can you tell me about these two rocks, blocks or books?

What can we measure for these two things?

What are the words we use for mass?

When would we want to measure mass?

Why would we want to measure mass?

How can I work out which bag is heavy?

What can you see when I hold a bag in each hand?

How could I draw a picture which shows which bag is heavy?

Check that students:

- can heft
- use language *heavy* and *light*
- can tell when objects are heavy, light or have about the same mass.

Discussion

What is the heaviest thing you can think of?

What is the lightest thing you can think of?

Mass 1.2 lesson ideas

Identification of attribute: compare and order objects by hefting

Knowledge and strategies

1. use comparative language: *heavier than*, *lighter than*, *heaviest*, *lightest*
2. predict which object would be heavier than, lighter than, about the same and explain reasoning
3. compare masses systematically and explain why a mass fits into a particular ordering

Bags in order

Given a number of identical bags or containers filled with different materials such as pasta, marbles, crumpled paper, nails, screws or beads. Students put them in order from lightest to heaviest. Given another bag they can decide where it should fit in the sequence.

Observe students's strategies, do they reorder all the bags or check the new bag against selected bags in the sequence?

Repeat with closed ice cream containers with different contents.

Outcomes

MES1.4
WMES1.2

Materials

opaque bags or containers with lids, variety of materials

Knowledge and strategies

1. use comparative language: *heavier than*, *lighter than*, *heaviest*, *lightest*
2. predict which object would be heavier than, lighter than, about the same and explain reasoning
3. compare masses systematically and explain why a mass fits into a particular ordering

Heavier than or lighter than? (see lesson plan)

Students are given three or four small items. (Different objects or different numbers of the same object). Students estimate and then order the objects by hefting. Record and discuss.

Extension: Check by dropping each item or group of items into suspended, knee-high stockings or long socks.

Outcomes

MES1.4
WMES1.4

Materials

small objects to order by mass, paper and pencils for recording

Extension: socks or stockings suspended (pegged to a line or hung on hooks)

Knowledge and strategies

1. use comparative language: *heavier than*, *lighter than*, *heaviest*, *lightest*
2. predict which object would be heavier than, lighter than, about the same and explain reasoning
3. compare masses systematically and explain why a mass fits into a particular ordering

What do you think?

Prediction game. Each child in a group has four objects.

Students predict whether their four objects are heavier, lighter, or about the same as another student's four objects. (feather, shoes, hats, cotton balls, leaves).

Outcomes	Materials	Knowledge and strategies
MES1.4 WMES1.1	collections to be compared, e.g. 5 pencils and 5 sticks, bags or containers for the items	1. use comparative language: *heavier than*, *lighter than*, *heaviest*, *lightest* 2. predict which object would be heavier than, lighter than, about the same and explain reasoning

Can I pull it?

Given three large objects on the floor, students estimate which is heaviest or lightest. Check the masses by tying a cord around each object and trying to pull it across the floor.

Outcomes	Materials	Knowledge and strategies
MES1.4 WMES1.1	three large objects of different masses, cord	1. use comparative language: *heavier than*, *lighter than*, *heaviest*, *lightest* 2. predict which object would be heavier than, lighter than, about the same and explain reasoning 3. compare masses systematically and explain why a mass fits into a particular ordering

Your turn to choose

Pairs of students select three items from a collection. Students estimate, then place the items in order of mass by hefting. Record the order. Students should be able to explain in pairs how the items were compared.

Outcomes	Materials	Knowledge and strategies
MES1.4 WMES1.3	Collection of items for students to make selections, pencils and paper	1. use comparative language: *heavier than*, *lighter than*, *heaviest*, *lightest* 2. predict which object would be heavier than, lighter than, about the same and explain reasoning 3. compare masses systematically and explain why a mass fits into a particular ordering

Mass 1.2 lesson plan

Identification of attribute: compare and order objects by hefting

Heavier than or lighter than?

Students are given three or four small items. (Different objects or different numbers of the same object). Students estimate and then order the objects by hefting. Record and discuss.

Extension: Check by dropping each item or group of items into suspended knee-high stockings or long socks.

Students should:

1. use comparative language: *heavier than*, *lighter than*, *heaviest*, *lightest*
2. predict which object would be heavier than, lighter than, about the same and explain reasoning.
3. compare masses systematically and explain why a mass fits into a particular ordering.

Grouping:

Step 1: whole-class discussion

Step 2: work in groups of two or three

Step 3: report back to the class.

Materials:

small objects to order by mass, paper and pencils for recording; Extension: socks or stockings suspended (perhaps pegged to a line or hung on hooks)

Outcomes

MES1.4 Compares the masses of two objects and describes mass using everyday language.

WMES1.4 Uses concrete materials and/or pictorial representations to support conclusions.

MASS

Activity

Step 1

- Introduce the activity as measuring mass.
- Revise the skill of hefting.
- Revise the vocabulary that will be used; write words on chalkboard.
- Introduce the task: students work with a partner or small group to order three or four objects by mass. Start by estimating. Remind students to record their measuring.

Extension:

Check the order by placing the objects into socks or stockings hung from a line or hook.

Questioning

What could I find out about these three toys?

How could I find the heaviest toy? The lightest toy?

What does it mean if I ask you to put the toys in order? (of mass) How could you do that?

What are the words you would use?

Which one of these do you think is the lightest? Why? How would you draw your measuring?

Extension:

What will you see happening if I drop a heavy toy into this stocking, and a light toy into the next stocking? What are the stockings showing us? Why?

Step 2

- Activity: Students take turns with their partner to heft each of the objects and then put these into order of mass.
- Students record the activity by drawing the objects and labelling. Students may be able to write a description of what they did.

Check that students:

- are correctly using language such as *heavy*, *heavier than*, *lighter than*
- can estimate first and explain
- can heft three objects and explain why they fit into an order.

Step 3

- Discuss the procedure used. Encourage students to think about estimation strategies, and why estimation is useful when measuring.

Discussion

What did you do today?

Were you able to estimate the heaviest and lightest? How?

What does your recording say?

Mass 1.3 lesson ideas

Identification of the attribute: compare masses using an equal-arm balance

Knowledge and strategies

1. know that an identical object on each side will balance
2. know that the side that goes down has the greater mass
3. demonstrate how to use an equal-arm balance to compare three or more objects and report the results

Everyone can be a balance

Students stand with their arms outstretched to simulate equal-arm balances. Teacher holds an object in each hand and asks students to predict and demonstrate what would happen to their arms if the objects were placed in their hands. The teacher places the objects in a student's hands to test the predictions.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.2	objects for demonstration	1. know that an identical object on each side will balance 2. know that the side that goes down has the greater mass

Using an equal-arm balance

Students need to be taught how to use a balance, e.g. stopping when just balanced, and discussing the idea of fractional masses, e.g. *It is heavier than two blocks but lighter than three blocks so its mass must be between two and three blocks. It might be two and a half blocks*. How to solve this situation can only be introduced at Level 3, as students need to understand that you could measure with a unit smaller than a block and work out how many blocks this would be.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.2	equal-arm balance, items to measure and the necessary number of blocks, rods, etc. to balance	1. know that an identical object on each side will balance 2. know that the side that goes down has the greater mass

Things on strings

Students find two identical objects and tie these on the ends of a piece of string. Suspend the string over a table or desk to balance the objects. Students explain and record their actions.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.1	Pieces of string objects to tie on string	1. know that an identical object on each side will balance 2. know that the side that goes down has the greater mass

Make the same mass (see lesson plan)

Make a mass (e.g. with modelling clay) that is the same as a given mass.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.2	equal-arm balance, material to be measured and additional material available to be used, e.g. modelling clay or dough, sand, rice	1. know that an identical object on each side will balance 2. know that the side that goes down has the greater mass 3. demonstrate how to use an equal-arm balance to compare three or more objects and report the results

What's your prediction?

Pairs of students compare three groups of items which have the same number, but different kinds of objects, such as five pencils, five cups and five interlocking blocks or three empty margarine containers, three blocks and three balls. Students predict first, then find which group has the greater mass by using an equal-arm balance.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.2	groups of objects for students to compare, pencils and paper	1. know that an identical object on each side will balance 2. know that the side that goes down has the greater mass 3. demonstrate how to use an equal-arm balance to compare three or more objects and report the results

Mass 1.3 lesson plan

Identification of the attribute: compare masses using an equal-arm balance

Make the same mass

Make a mass (e.g. with modelling clay) that is the same as a given mass.

Students should:

1. know that an identical object on each side will balance
2. know that the side that goes down has the greater mass
3. demonstrate how to use the equal-arm balance to compare three or more objects and report the results.

Grouping:

Step 1: whole-class discussion

Step 2: work in groups of two or three

Step 3: report back to the class.

Outcomes

MS1.4 Estimates, measures, compares and records the masses of two or more objects using informal units.

WMS1.2 Uses objects, diagrams, imagery and technology to explore mathematical problems.

Materials:

equal-arm balance, material to be measured and additional material available to be used, e.g. modelling clay or dough, sand, rice

MASS

Activity

Step 1

- Introduce the activity as measuring *mass*.
- Note: these notes assume that students have been introduced to the concept and use of the equal-arm balance, perhaps through the whole-class lesson "Everyone can be a balance".
- Revise how to use an equal-arm balance.
- Introduce the activity and discuss what to do, and how to record what happened.
- Show the given mass to the students (an object or an amount of modelling clay or dough) and demonstrate the materials that could be used to balance this.

Step 2

- Students work in pairs or small groups with an equal-arm balance to make a mass.
- Students record what they did to make an equal mass.

Step 3

- Discuss difficulties with using the equal-arm balance, materials or items that are not appropriate for this instrument.
- Discuss conservation of mass: use two identical balls of modelling dough, and break one ball up into a number of pieces. Are they still the same mass?

Questioning

What are the words we use to talk about mass?

What will an equal-arm balance help you to do? How does it work?

What will I see if my mass of modelling clay, dough or sand is heavier than the mass that the teacher gave me? How will I fix it?

How will I know when the masses are the same?

How could I do the recording?

What words will I need?

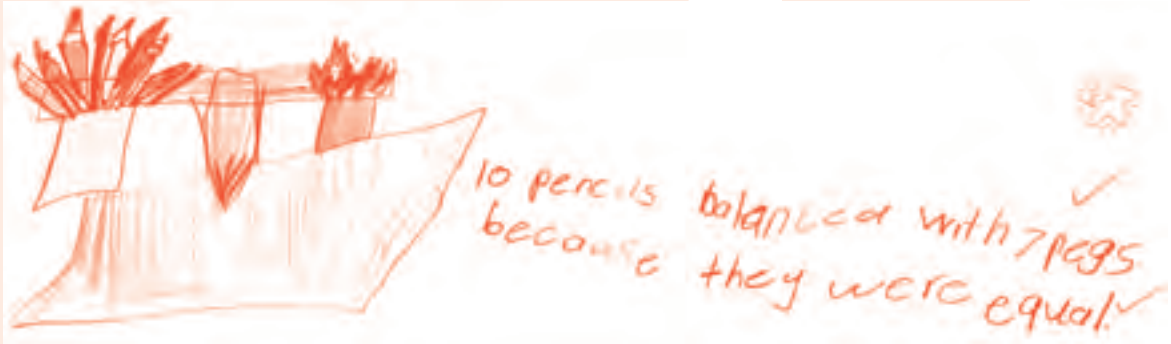
Check that students:

- can use the equal-arm balance
- can explain why one side goes up or down
- can explain what happens when the masses are the same.

Discussion

Was it difficult to get the masses exactly the same?

If you have two things with the same mass, do they have to be the same size?



Mass 2.1 lesson ideas

Informal measurement: choose appropriate units and use them to measure a mass

Knowledge and strategies

1. find out how many identical units will balance a given mass
2. suggest appropriate units and explain why some units would be better than others
3. explain that spatial structure is not important for mass and that round objects such as marbles are suitable for measuring mass

Make another bag (see lesson plan)

The teacher displays a bag with some blocks in it. Students make a bag that has the same mass by filling with blocks and then hefting the two bags.

Students find the mass of their bag by choosing appropriate units and measuring on an equal-arm balance. The measuring process and results are recorded, including a comment on the choice of units.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.3	equal-arm balance, two bags (one prepared with blocks or equipment), units suitable for comparing mass, pencils and paper	1. find out how many identical units will balance a given mass 2. suggest appropriate units and explain why some units would be better than others 3. explain that spatial structure is not important for mass and that round objects such as marbles are suitable for measuring mass

What do you think?

Students predict how many of a given unit it will take to balance an object and then check by measuring. Students record their estimate and measure using their own words and format.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.4	equal-arm balance, item to be measured, blocks or similar units for measuring mass, pencils and paper	1. find out how many identical units will balance a given mass 2. suggest appropriate units and explain why some units would be better than others

Make a balance

In pairs, students make a balance using a coathanger, stockings or socks and pegs. Students choose appropriate units to balance an object which is suspended in the sock. The mass of the object should be estimated and recorded, before the item is measured. Students count and record their measure.

Outcomes

MS1.4
WMS1.2

Materials

coathangers, socks or stockings, pegs, pencils and paper, items to measure

Knowledge and strategies

1. find out how many identical units will balance a given mass
2. suggest appropriate units and explain why some units would be better than others

A cup of rice

Pairs of students choose suitable measuring units to find the mass of a cup of rice. Students record the mass and state why they chose the units.

Class discusses the results and compares the units which were chosen. Some units may have a greater or smaller volume than other units.

Outcomes

MS1.4
WMS1.2

Materials

equal-arm balance, one cup of rice in a closed container, choice of units to measure mass, pencils and paper for each pair of students

Knowledge and strategies

1. find out how many identical units will balance a given mass
2. suggest appropriate units and explain why some units would be better than others
3. explain that spatial structure is not important for mass and that round objects such as marbles are suitable for measuring mass

Does it balance?

Students are given a choice of objects they might use to find the mass of different objects using an equal-arm balance.

It is important that students are given tasks that emphasise different volumes can have the same mass and vice versa to differentiate these two concepts.

Outcomes

MS1.4
WMS1.5

Materials

equal-arm balance, items to measure access to blocks, rods, etc. to balance

Knowledge and strategies

1. find out how many identical units will balance a given mass
2. suggest appropriate units and explain why some units would be better than others
3. explain that spatial structure is not important for mass and that round objects such as marbles are suitable for measuring mass

Mass 2.1 lesson plan

Informal measurement: choose appropriate units and use them to measure a mass

Make another bag

The teacher displays a bag with some blocks in it. Students make a bag that has the same mass by filling with blocks and then hefting the two bags.

Students find the mass of their bag by choosing appropriate units and measuring on an equal-arm balance. The measuring process and results are recorded, including a comment on the choice of units.

Students should:

1. find out how many identical units will balance a given mass
2. suggest appropriate units and explain why some units would be better than others
3. explain that spatial structure is not important for mass and that round objects such as marbles are suitable for measuring mass.

Grouping:

Step 1: whole-class discussion

Step 2: work in groups of two or three

Step 3: report back to the class.

Outcomes

MS1.4 Estimates, measures, compares and records the masses of two or more objects using informal units.

WM1.3 Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols.

Materials:

equal-arm balance, two bags (one prepared with blocks or equipment), blocks or similar units suitable for comparing mass.

MASS

Activity

Step 1

- Introduce the activity as the measurement of *mass*.
- Explain the activity: Students are given a bag and have to make a second bag with the same mass by trial and error in adding a material, and hefting. Discuss what materials could be placed into the second bag.
- Discuss how to measure the mass of each bag so that students can check how close they were to making equal masses. (Use an equal-arm balance and identical units to measure each mass.)
- Record the measured mass of each bag and compare the results of the two bags.

Step 2

- Students work with a partner or small group to: make a second mass, heft both masses and adjust their second mass; find the mass of each bag, using identical units. Record and compare the two masses.

Step 3

- Discuss the measurements and the ability to make a matching mass.
- Discuss how to heft accurately.

Questioning

What could I find out about this bag? What measurements could I make?

If I asked you to make another bag with the same mass, how would you compare the masses quickly?

How would you compare the masses very accurately?

What kind of units would you use for this?

Does the second bag need to have the same volume as the first bag? Why?

Any advice for choosing and using units?

Do I have to pack the units or make sure they cover? Why?

What could the recording look like?

Check that students:

- are using identical units
- can explain why they chose units
- can explain that units of mass can take up more or less space (volume), and that gaps, covering and packing don't matter.

Discussion

How close were your two masses?

Did you expect to be exactly right? Why?

Were some materials better to use than others? Why?

Mass 2.2 lesson ideas

Informal measurement: compare and order masses using identical units for each mass

Knowledge and strategies

1. choose identical units to measure each mass
2. explain that the heavier mass has more units
3. estimate the number of units and explain the estimation strategy
4. know that mass is conserved if rearranged

Which is heavier?

Estimate then find which of two objects is heavier (but the students are not allowed to heft them or to put them on the balance together).

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.1	equal-arm balance, two objects to compare, blocks or similar units suitable for measuring mass	1. choose identical units to measure each mass 2. explain that the heavier mass has more units 3. estimate the number of units and explain the estimation strategy

Heaviest pencil case (see lesson plan)

Work in groups of three or four to estimate, then measure whose pencil case is heaviest by measuring the mass of each pencil case with blocks (teddies, marbles etc.). Ensure that the same units are selected for measuring. Record in order of mass.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.3	equal-arm balance, two objects to compare, blocks or similar units suitable for measuring mass	1. choose identical units to measure each mass 2. explain that the heavier mass has more units 3. estimate the number of units and explain the estimation strategy

Has to be the same mass

My mystery object can be balanced by five blocks. Find or make three objects that would have the same mass. How can you prove you are correct? Students record their trials and answers.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.4	equal-arm balance, blocks, access to objects around the room, pencils and paper	1. choose identical units to measure each mass 2. explain that the heavier mass has more units 3. estimate the number of units and explain the estimation strategy

Mystery boxes

Students are given three or four identical opaque boxes, such as margarine containers, which each hold one item. Students place the containers in order by mass and record their prediction of what the contents might be.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.4	equal-arm balance, four containers for each pair or small group of students, pencils and paper	1. choose identical units to measure each mass 2. explain that the heavier mass has more units 3. estimate the number of units and explain the estimation strategy

No more gaps

Discuss and predict the mass of the same quantity of a specific object in two different structures. For example:

Does a flat have the same mass as 100 shorts?

Do ten loose popsticks have the same mass as a bundle of ten sticks? (ten loose interlocking blocks and a rectangular prism of ten blocks.)

Measure each quantity to find the mass.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.4	shorts, longs, interlocking blocks, popsticks etc.	1. choose identical units to measure each mass 2. explain that the heavier mass has more units 3. estimate the number of units and explain the estimation strategy 4. know that mass is conserved if rearranged

Mass 2.2 lesson plan

Informal measurement: compare and order masses using identical units for each mass

Heaviest pencil case

Work in groups of three or four to estimate, then measure whose pencil case is heaviest by measuring the mass of each pencil case with blocks (teddies, marbles etc.). Ensure that the same units are selected for measuring. Record in order of mass.

Students should:

1. choose identical units to measure each mass
2. explain that the heavier mass has more units
3. estimate the number of units and explain the estimation strategy.

Grouping:

Step 1: whole-class discussion

Step 2: work in groups of two or three

Step 3: report back to the class

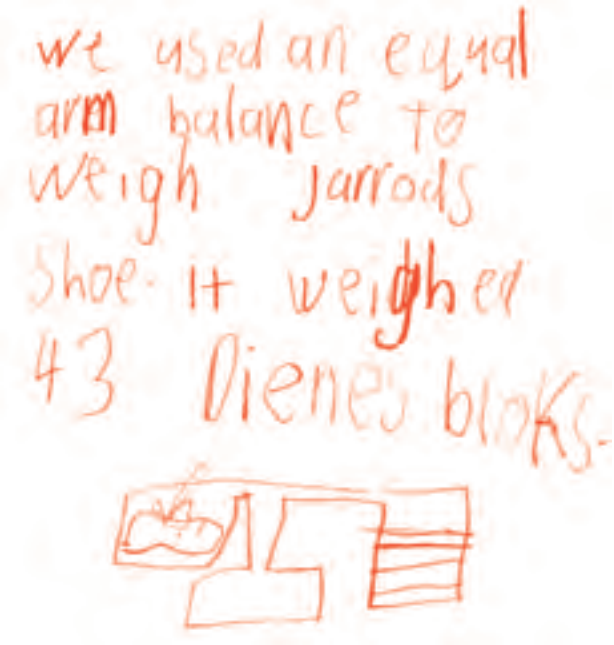
Outcomes

MS1.4 Estimates, measures, compares and records the masses of two or more objects using informal units.

WM1.3 Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols.

Materials

coathangers, socks, stockings or bags, pegs, recording paper, units for measuring



MASS

Activity

Step 1

- Introduce the lessons as measurement of *mass*.
- Discuss how small groups of students could order their pencil cases by mass, without putting more than one pencil case on an equal-arm balance at any time.
- Discuss how to measure the mass of each pencil case, using an equal-arm balance.
- Discuss how to record the results and write a sentence explaining which pencil case had the greatest mass, and why.

Questioning

How could I measure this pencil case? What units could I use for each measurement? (length, area, volume, mass)

How can I compare the mass of two pencil cases, without putting both pencil cases on a balance at the same time?

What could I use as units?

Will all members of my group use the same units? Why?

Could we use a mix of units to measure? Why?

How would we write our results?

What words could be used in my recording?

Step 2

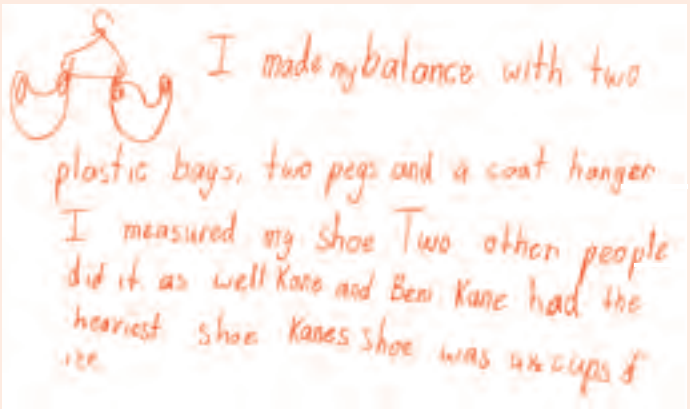
- Students work within groups to order the pencil cases.
- Students estimate first, then find the mass of each pencil case, record, and compare the results.
- Write an explanation of how the answer was found.

Check that students:

- are using identical units for both measures
- have ensured that the balance is working correctly before measuring
- understand that the heaviest pencil case has more units
- count and record units accurately.

Step 3

- Discuss the process of choosing and using units.



Discussion

Who estimated the correct answer?

Whose estimation was close to the actual measurement?

Was the biggest pencil case the heaviest pencil case? Why?

Mass 3.2 lesson ideas

Relationship between units: explain the relationship between unit size and number of units required to balance a mass

Knowledge and strategies

1. explain the relationship between the unit size and the number of units
2. express the same mass in terms of different-sized units
3. know that measurement techniques must be consistent and precise

Work it out (see lesson plan)

Teacher or student measure the mass of an object in blocks (e.g. using 10 blocks) Using this measure, students predict how many of another unit, e.g. how many ones, would be needed to balance the object. Record the estimate and calculation before using a balance to check.

Outcomes

MS1.4
WMS1.4

Materials

equal-arm balance, object, blocks used to measure mass, smaller units to work with

Knowledge and strategies

1. explain the relationship between the unit size and the number of units
2. express the same mass in terms of different-sized units
3. know that measurement techniques must be consistent and precise

Heavier or lighter?

I have a bag with some blocks in it. Use given unit, such as marbles, to balance my bag. Do you think a marble is heavier or lighter than a block (a lot heavier or just a little)? Explain or write your answer.

Outcomes

MS1.4
WMS1.5

Materials

equal-arm balance, small bag of blocks, marbles or similar units, pencils, paper

Knowledge and strategies

2. express the same mass in terms of different-sized units
3. know that measurement techniques must be consistent and precise

Let's be accurate!

Teacher models and whole-class discussion of technique, followed by student investigation in pairs or small groups. Class finds the mass of a given object using MAB materials. Commence by comparing with blocks. Students suggest how to measure more accurately by using smaller units (flats, then longs, then shorts) Discussion should occur at each decision point.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.3	MAB materials, objects to measure, equal-arm balances	1. explain the relationship between the unit size and the number of units 2. express the same mass in terms of different-sized units 3. know that measurement techniques must be consistent and precise

Solve the mystery

My mystery object can be balanced by (for example) three blocks and five pencils. How many pencils would it take to balance it? Predict and check. Record how you worked it out.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.4	equal-arm balance, blocks, new pencils or another unit, additional pencils, access to other units, pencils and paper	2. express the same mass in terms of different-sized units 3. know that measurement techniques must be consistent and precise

Work out how many

My mystery object has the same mass as two eggs. How many blocks would I need to balance it, (only give them one egg). Students should check that they are correct and explain or record their working.

Outcomes	Materials	Knowledge and strategies
MS1.4 WMS1.2	equal-arm balance, object, one unit which has been used, suitable smaller units, pencils and paper	1. explain the relationship between the unit size and the number of units 2. express the same mass in terms of different-sized units 3. know that measurement techniques must be consistent and precise

Mass 3.2 lesson plan

Relationship between units: explain the relationship between unit size and the number of units required to balance a mass

Work it out

Teacher or student measures the mass of an object in blocks (e.g. using ten blocks) Using this measure, students predict how many of another unit, e.g. how many ones, would be needed to balance the object. Record the estimate and calculation before using a balance to check.

Students should:

1. explain the relationship between the unit size and the number of units
2. express the same mass in terms of different-sized units
3. know that measurement techniques must be consistent and precise.

Grouping:

Step 1: whole-class discussion

Step 2: work in groups of two or three

Step 3: report back to the class.

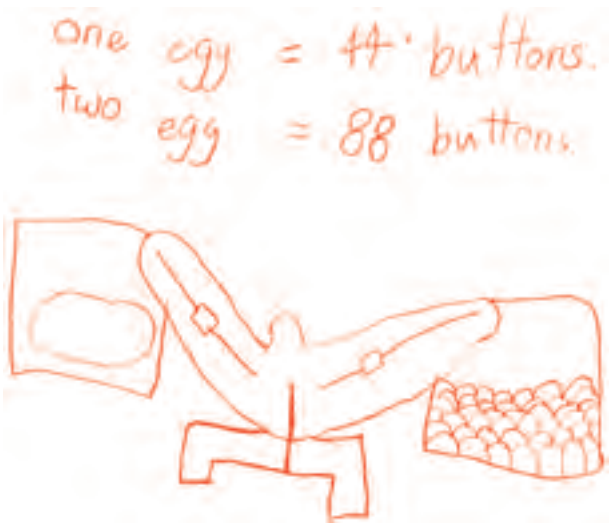
Outcomes

MS1.4 Estimates, measures, compares and records the masses of two or more objects using informal units.

WM1.4 Supports conclusions by explaining or demonstrating how answers were obtained.

Materials:

equal-arm balance, object, blocks used to measure mass, smaller units to work with, paper and pencil for recording



MASS

Activity

Step 1

- Introduce the lesson as the measurement of *mass*.
- Demonstrate that the mass of the object has already been measured, and is ten large blocks.
- Ask students how they would find the mass in smaller units, if they were given only one of the large blocks (and not the object).
- Discuss the measuring equipment that may need to be used, and how to record the calculations and results.
- Discuss how the calculations may be made.

Questioning

How could I measure this block? What would I use as units? (Discuss length, area, volume, mass.)

What does it mean when I say that this object has a mass of ten blocks?

If I tell you that this object has a mass of ten large blocks, how could you find the mass in small blocks, if you had just one of the large blocks and your small blocks?

How would you estimate the answer?

What sort of working out would you do?

What measuring would you have to do?

Step 2

- Students work with a partner or small group to record their estimate, calculate the number of small blocks and then check the result with an equal-arm balance.

Check that students:

- have made careful estimates
- understand that they will need more of the smaller units
- can explain what they are doing, and why.

Step 3

- Discuss how the answer was found.
- Discuss what happens to the answer when smaller units are used.
- Discuss different methods used by groups or pairs.

Discussion

Was your answer smaller or larger than the number of large blocks? Why?

When might it be a good idea to use smaller units for measuring mass? Why?

ISBN 073138258-7