 Year 12 Mathematics Standard 1

Unit title: MS-A4 Types of Relationships 

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Duration: 2 weeks

Rationale

A knowledge of algebra enables the modelling of a problem conceptually so that it is simpler to solve, before returning the solution to its more complex practical form. A study of algebra is important in developing students’ reasoning skills and logical thought processes, as well as their ability to represent and solve problems. The ability to interpret and critically evaluate information that is presented in graphical form will provide students with important skills they will use when making decisions in the future.

Topic focus

The principal focus of this subtopic is the graphing and interpretation of relationships, and the use of simultaneous linear equations in solving practical problems.

Students develop their ability to communicate concisely, use equations to describe and solve practical problems, and use algebraic or graphical representations of relationships to predict future outcomes.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

Prior knowledge required

* Substitution into a formula
* Creation of a table of values
* Plotting points on the Cartesian plane

Language considerations

* Break-even point
* Cost
* Equation
* Exponential
* Exponential model
* Graph
* Linear
* Linear equations
* Line of best fit
* Model
* Non-linear
* Non-linear model
* Physical phenomena
* Point of intersection
* Profit
* Revenue
* Simultaneous linear equations
* Sketch
* Straight-line graph

Outcomes

* uses algebraic and graphical techniques to critically evaluate and construct arguments in a range of familiar and unfamiliar contexts MS1-12-1
* represents the relationships between changing quantities in algebraic and graphical forms MS1-12-6
* chooses and uses appropriate technology effectively and recognises appropriate times for such use MS1-12-9
* uses mathematical argument and reasoning to evaluate conclusions, communicating a position clearly to others MS1-12-10

Assessment

Pre-test – A pre-test on creating tables of values by substituting into a rule, and then plotting points and graphing linear equations would be useful to determine students’ skills before starting this topic. Their performance on this test would determine how much revision of these skills you would need to complete before starting this topic.

Summative assessment – ‘How to build a disease’ Investigative Project

* This project has been designed to try and reduce plagiarism by asking students to generate their own data.

| Content | Teaching and learning strategies and evidence of learning | Resources |
| --- | --- | --- |
| A3.1 Simultaneous linear equations | Teachers may need to revise techniques for creating tables of values and plotting linear graphs before undertaking the rest of the unit. | N/A |
| * solve a pair of simultaneous linear equations graphically, by finding the point of intersection between two straight-line graphs, using technology **Paperclip icon**  Information and communication technology capability icon
 | Key ideasStudents need to understand that the point where the two graphs meet have to satisfy both equations.Teaching strategiesDesmos or Geogebra may be used to enter equations for straight lines and draw the graphs so that students can simply read off the point of intersection. The activities below provide a practical context for the solution of simultaneous equations.Teaching activities* ‘Racing Dots’ is a good introductory activity as it introduces a real life context for solving simultaneous equations and revises a number of skills that students will need later in the topic.
	+ Students sign into the Desmos activity, ‘Racing Dots’. They are asked to predict where two dots will meet and how confident they are of their answer.
	+ They are then allowed to see their classmates’ estimates and teacher leads a discussion as to what data we need to make our estimates more accurate (for example, speed of the dots).
	+ Students are then given the position of the dots after 4 seconds and asked to revise their original estimate. Teacher could at this point remind students how to find the speed of an object.
	+ The program then shows students three methods to find the answer; table of values, graphically and algebraically. (Although algebraic methods are not a part of this course, it doesn’t hurt to remind students of other methods).
* A good follow up activity to the previous one is to look at different mobile phone contracts. It provides an opportunity to revise linear equations and substitution.
	+ Use the Gynzy website to write an equation for two different phone plans. Graph each equation and then discuss which plan would be better over different periods of time. It is important to point out why the graphs start at zero and that the lines don’t extend to the left of the y-axis.
	+ The website then demonstrates how the point of intersection satisfies both equations and goes on to give numerous examples of graphing two equations and finding the point of intersection.
	+ The activity finishes with a quiz that returns to the phone plan question at the start of the lesson.

After these initial introductory activities, students will need to practice drawing graphs and reading off the point of intersection. Students should be proficient at this before moving onto practical contexts. | [Desmos](https://www.desmos.com/calculator)[Geogebra](https://www.geogebra.org/graphing)[Racing Dots](https://teacher.desmos.com/activitybuilder/custom/56d139907e51c4ed1014b51f)* In this activity, students predict where a pair of moving dots will meet by using tables, graphs, and/or equations. While students can use any of those representations to solve the challenge, the activity was designed with an eye toward solving systems of linear equations via substitution.

[Gynzy](https://api.gynzy.com/en/#!/lesson_plans/224)* Requires sign up
* Introduces solving simultaneous equations within a mobile phone context. Teacher-directed lesson through interactive whiteboard lesson.
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| * develop a pair of simultaneous linear equations to model a practical situation **AAM** **Paperclip icon** Critical and creative thinking icon  Information and communication technology capability icon
* solve practical problems that involve finding the point of intersection of two straight-line graphs, for example determine and interpret the break-even point of a simple business problem where cost and revenue are represented by linear equations **AAM** **Paperclip icon** Work and enterprise icon
 | Key ideasStudents look at a range of practical situations that can be solved by using simultaneous equations. They should be given the opportunity to develop equations from the information given. Students need to understand that the point where the two graphs meet is the ‘break-even point’.Teaching activities* Supply and demand or profit and loss scenarios are commonly solved using simultaneous equations. Using the scenario of a cake-shop owner selling muffins for $2.50 each with costs of $1 to make each muffin and $300 for the equipment needed to make the muffins, the teacher should help the students to develop equations for income and expenditure before graphing the equations and determining the break-even point.
	+ As per the previous examples, students should discuss what is happening in different parts of the graph and why it is not realistic to extend the graph to the left of the y axis.
* ‘Stacking Cups’ video
	+ Students are to watch the ‘Stacking Cups’ video. They are then asked to guess how many cups are required for the stacks to be equal.
	+ The teacher should then discuss with students what information we require to be able to solve the problem. For example, the height of the cups and the width of the lips. The pictures underneath the video in Act 2 show the height of each type of cup and the width of the lip.
	+ At this point the teacher should help the students to develop an equation for the height of each type of cup based on the number of cups.
	+ Students should then graph each equation to determine the answer. The video in Act 3, shows the correct answer of 7.
	+ Teacher and students could then brainstorm other questions that can be asked about the cups. For example, how many of each cup is required to reach a certain height? A number of different questions are given at the bottom of the video.
* Teacher sets the scene of the tortoise and hare race. The tortoise takes off at 0.5km per hour. The hare, being cocky, leaves 4 hours later at 2km/h. Who will win the race?
	+ Students should be given the opportunity to try and solve the problem themselves. Some discussion of the different methods used should follow.
	+ The teacher could then help the students to develop equations or tables of values so they can graph the equations and find a solution. This is a good opportunity to practice converting decimal time to hours and minutes or minutes and seconds.
	+ Students can then find the point where they catch up, but who actually wins? Need to discuss how long the race was and who would win for different race lengths.
	+ The Youtube video shows how the equations are developed and how to read and interpret the solution. It would be best to show this after the students have tried to find the solution for themselves.
 | [Stacking Cups](http://www.101qs.com/1897)* Stacking two different types of disposable cups. How many cups does it take for the heights to be equal?

Youtube – [Systems of linear equations in two variables](https://www.youtube.com/watch?v=75m60SxFfJg)* This video shows how simultaneous equations can be used to solve this age-old problem.
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| A3.2 Graphs of practical situations | N/A | N/A |
| * sketch the shape of a graph from a description of a situation, for example the time passed and the depth of water in different shaped containers, or the speed of a race car as it moves around different shaped tracks Critical and creative thinking icon
 | Key ideasStudents must realise that not all situations can be modelled using a linear graph. They will look at, in particular, quadratic and exponential equations as better models in particular situations.Teaching activities* The ‘Graphing Stories’ website provides a number of different real life scenarios for students to practise graphing. There are a variety of different graph types. The website has a handout that teachers can print which has axes already drawn and an appropriate time scale.
	+ Students initially watch the event and then the video replays it at half speed so students get a better idea of what is happening.
	+ At this point, students could make a first attempt at drawing a graph to represent the action.
	+ The teacher should then replay the video and point out any important events such as change of direction or change of speed or height.
	+ Students should then amend their graph if necessary.
	+ The video then shows the correct graph.
	+ The teacher should discuss any differences between student graphs and the correct graph.
	+ ‘Bum height off ground’ produces a decreasing graph that has linear elements with stationary points
	+ ‘Distance from bench’ produces a periodic graph
	+ ‘Height of tennis ball’ produces a parabolic graph that decreases
	+ ‘Height of waist off ground’ also produces a periodic graph
* Youtube video, ‘On board F1 GP Australia 2013 (camera car)’ show the speedometer of a race car as it goes around a track. As above, students could graph the speed of the car over the first minute.
* The ‘Filling Glasses’ and ‘Container Filling’ activities are a good follow up to the previous activity and reinforce the idea that not all graphs are linear.
	+ Students need to notice that the fluid level rises more quickly in a narrow glass than in a wide glass and they need to identify the other variables which determine the shape of each line graph.
	+ Both activities involve students matching different shaped containers to their appropriate graphs. It shows many different types of graphs without introducing terms such as exponential.
	+ After the students have played with the interactives for a while, it would be timely to stop the class and discuss with students why different shaped containers give different graphs and what is causing a graph to become steeper or flatter.
	+ Students can then use this shared knowledge to aid them in completing the rest of the activities.
	+ If teachers do not have access to technology they could bring in a range of different shaped containers and have students fill them with water, taking notice of how quickly the water level rises in different parts of the container.
 | [Graphing Stories](http://graphingstories.com/)Youtube – [On board F1 GP Australia 2013 (camera car)](https://www.youtube.com/watch?v=8uPphTGrhNM)[Filling Glasses](http://www.scootle.edu.au/ec/viewing/L759/index.html)* A number of different activities on the Scootle website examine the relationship between the shape of a glass and the time taken to fill it with juice. This learning object is one in a series of five objects.

[Container Filling](https://www.geogebra.org/m/S4Yc2fda)* Interactive demo of the depth-time graphs for shapes of containers being filled with liquid. Students are able to manipulate the shape of containers.
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| * construct a graph from a table of values both with and without technology  Information and communication technology capability icon
	+ use values of physical phenomena, eg the growth of algae in a pond over time, or the rise and fall of the tide against a harbour wall over time to plot graphs and make predictions
 | Key ideasStudents are introduced to more examples of non-linear graphs that can be used to model real life situations.Teaching activities* Using a copy of historic tide times from the Bureau of Meteorology, students can graph tide times versus tide heights over a number of days.
	+ They could then compare their graphs with those on the Willy Weather site. Discussion points are around the cyclic nature of the graph. How often does the graph repeat? What is the highest point? What is the lowest?
* In the ‘Lake Algae’ activity, students study a species of algae that is doubling in area every day.
	+ Have students discuss questions such as how long it would take to completely cover the lake, or, if 99% of the algae was removed, how long would it take before the lake would again have to be cleared?

Key ideasStudents are introduced to exponential graphs and the change in gradient from the start of the graph to the end.Teaching activities* Pose the question to students – ‘Can folding a piece of paper 45 times get you to the moon?’ Ask for their opinions.
	+ Create a table of values and start with 0 folds giving 1 thickness, 1 fold giving a thickness of 2, and so forth.
	+ Have students draw the graph either by hand or by using a graphing program to see how quickly the thicknesses increase.
	+ Have students predict how thick the paper would be after 45 folds.
	+ Assuming the paper is 0.001mm thick initially, how thick will it be after 45 folds? How close is this to the moon?
	+ Is this model practical in real life? How many times can students fold a piece of paper? Have students fold a piece of A4 Reflex paper. Can they fold this more or less times than a sheet of tissue paper of the same size?
	+ Students could then watch the Ted Ed lesson on the power of exponential growth.
* Tell students the story of rice on a Chessboard.
	+ Students could then discuss how much rice they think the inventor would have once the king had finished completing the chessboard.
	+ Have students draw up a table of values showing the number of grains of rice on each square and a further column showing the total number of grains.
	+ Discuss how quickly the numbers grow.
	+ The students could then count how many grains of rice it takes to fill a small container in order to determine how much room would be required to store the inventor’s rice.
 | [Tide Times (Bureau of Meteorology)](http://www.bom.gov.au/oceanography/projects/ntc/nsw_tide_tables.shtml)[Australia Tide Times (Willy Weather)](https://tides.willyweather.com.au/)[Lake Algae](https://www.illustrativemathematics.org/content-standards/tasks/533)[How folding paper can get you to the moon](https://ed.ted.com/lessons/how-folding-paper-can-get-you-to-the-moon)* Ted-Ed lesson (video) examining the power of exponential growth. Includes discussion and questions.

[The rice and chessboard story](http://www.dr-mikes-math-games-for-kids.com/rice-and-chessboard.html) |
| * determine the best model (linear or exponential) to approximate a graph by considering its shape, using technology where appropriate **AAM** **Paperclip icon** Critical and creative thinking icon  Information and communication technology capability icon
* identify the strengths and limitations of linear and non-linear models in given practical contexts **AAM** Critical and creative thinking icon
 | Key ideasOn advice from NESA, the only non-linear function that needs to be explicitly taught with regards to construction and interpretation is the exponential function. Students may be required to interpret other non-linear functions (for example, find a point on the graph of the function) such as quadratic functions but they do not need to know that it is a quadratic function or hyperbolic function, and so on, or the general features of each.Teaching strategiesThis is a good opportunity to revisit simple interest (linear) and compound interest (exponential) as well as straight line depreciation (linear) and declining balance depreciation (exponential).Teaching activities* In this twist on a classic activity, students compare linear and exponential growth in the context of daily payments. One plan increases by $100 each day, while another starts at $0.10 and grows by doubling the previous day’s payment.
	+ This activity can be done by hand if there is no access to technology. Students draw up a table of values in their book and then graph the two models on the same graph.
	+ Students are asked to consider which method of payment is better and to then extend their thoughts to consider which payment is better over different lengths of time.
	+ The connection between table of values and graphs is reinforced.
	+ This activity is appropriate for students who have studied linear functions but may not have any experience with exponential growth. With that in mind, it makes a great first activity in an exponential functions unit.

Key ideasLinear graphs are not appropriate in all situations.Teaching activities* Although quadratics are not mentioned specifically in the syllabus, this activity is useful as it reinforces the idea that linear graphs are not appropriate in all situations. It does not assume any knowledge about quadratics, but demonstrates why a linear model is not appropriate.
	+ Students complete the ‘Will it hit the hoop?’ activity as an introduction to parabolas. It starts by asking students to fit a straight line through some points that indicate the position of a basketball moments after being thrown.
	+ It then demonstrates the limitations of this model as it assumes the ball will keep rising.
	+ Students are introduced to parabolas and are asked to drag points to make the parabola model the path of the basketball.
	+ Students use these parabolas to predict whether or not the ball will pass through the hoop.
	+ The last few extension questions introduce students to the idea of the vertex representing the maximum height of the ball. (This would be extension only, and is not necessary for this course)
 | [Avi and Benita’s repair shop](https://teacher.desmos.com/activitybuilder/custom/56c7457e11c7724106e683b1)Will it hit the hoop?* In this activity, students predict whether various basketball shots will go through the hoop, and then model these shots with parabolas to check their predictions.
* Students use movable points to model in this activity, and do not need to be familiar with the symbolic forms of quadratic functions in advance.
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Reflection and evaluation