 Induction

| Syllabus element | Teaching ideas | Teaching resources |
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| Mathematical Induction: Introducing the method | Mathematical Induction is typically taught late in the teaching programme as the application of this method requires students to be fluent in algebraic manipulation including* factorisation (with non-numeric factors)
* manipulation of indices, and
* substitution of non-numeric, multiple term values

In addition, students need to be comfortable with:* sigma notation,
* factorial notation and manipulation,
* combinatorial notation, and
* the concepts of inequality and expressions vs equations

It is useful to revise these skills before introducing induction.Since the method of mathematical induction is a fairly straight forward process, students can be introduced to the process by:* Narration (the domino effect/demonstration)
* Step-by-step instructions
* Scaffolded proofs
* Demonstration
* Extensive feedback

This allows students to grasp the concepts without overwhelming them initially.It is important that once students have become comfortable with the process of induction proofs, common misconceptions and issues are addressed through extensive feedback from the teacher. Mathematical Induction proofs tell a story and just like any good story they need to have a beginning, middle and end with no leaps in logic that the reader can’t follow. Any assumptions made, need to be explicitly stated and ‘wrapped up’ at the end. | [The domino effect is explained on the Maths is Fun website](https://www.mathsisfun.com/algebra/mathematical-induction.html).[An explanation of the steps (and why they are all essential) is available at the Purple Math website](http://www.purplemath.com/modules/inductn.htm).[Teachers can demonstrate induction themselves or utilise Wootube](https://www.youtube.com/playlist?list=PL5KkMZvBpo5Bf6wS0saU1yNvDxBWN5o9j), which has an extensive list of all the different types of induction. |
| Another metaphor for Induction: The Bridge in the mist | Would you walk across the bridge? What gives you confidence it is a sound structure?Although we can’t see the end:* We see the bridge is firmly anchored at the start
* We look ahead at a piece that is clearly functioning

and we see the piece next to it is also holding up |  |
| Mathematical Induction - types | Mathematical Induction questions are broadly categorised into the following types:* Sums of series
* Divisibility
* Inequality
* Formula for
* Proof of a calculus result (typically differentiation for Extension 1 students, integration by parts for Extension 2 students)
* Factorisation formula.
* Proof of geometric formula, identities etc.

Examples of all these types should be demonstrated to students so that teachers can highlight tips and tricks that will help them to progress in their proofs. Comments from HSC markers are a great source of these tips. Some include:* Making a specific term the subject of the equation helps for subsequent substitutions.
* You must state that an assigned variable is an integer when doing divisibility proofs.
* Students need to be diligent in avoiding transcription errors.
* Factorise whenever possible.
* Good practice is to write out the statement to be proven.

In cases where substitution doesn't work, remember that  can be used to illustrate divisibility as well. This is because if two numbers have a common factor, their difference will have the same common factor.  | [Worked examples of all types are given in Helen Bush’s article](http://hsc.csu.edu.au/maths/ext1/math_induction/179/mathematicalinduchbush2.pdf). |
| Mathematical Induction: Reducing misconceptions | Comments from HSC markers outline common misconceptions including:* Students know to use the assumption but do not manipulate the resulting expression to arrive at the final step.
* Students treat the proof as an equation and work on both sides.
* Students don’t know which base case to use, especially when it is not  or when it is in inequality (e.g.).
* Students don’t realise that in induction problems involving a series of terms, it is the sum of terms in the LHS that is being compared to the RHS, and not the general term
* Students don’t understand the difference between proving an inequality and solving an inequation.
* Students must show that the base case is true, not just state it.
* While students can state that the proof is true for, they must address the validity of this assertion at some point in the proof.
* Students won’t get marks for just writing out the structure of a mathematical induction proof with no attempt at the proof.

By explicitly drawing attention to these issues, teachers can guide students on these points so as to avoid such errors. | Utilise the [Markers Comments from past papers](http://www.boardofstudies.nsw.edu.au/hsc_exams/) for tips and misconceptions held by students. |
| More resources about induction and ways to teach it | The AMSI “Maths Delivers” project has an excellent paper on Induction. | [An AMSI publication on ‘Proofs by induction’](http://www.amsi.org.au/teacher_modules/pdfs/Maths_delivers/Induction5.pdf) |
| No magic incantation required | Markers at the MANSW HSC Feedback day have been very explicit in the last few years: students are **not**required to write a long “magic incantation” sentence at the conclusion of their induction proof. Provided the proof is well set out, clearly showing the steps of the inductive proof, it is sufficient to conclude with the statement ‘Hence true for all values of n by mathematical induction’. There are no marks allocated for the closing sentence. |  |
| Teacher background: Inductive proofs are actually proofs by deduction! | The process used in Proof by Mathematical Induction, as done by high school students is a **deductive**proof - clearly following a series of deductive reasoning. The “induction” only comes in when we appeal to the principle (or axiom) of mathematical induction. So we build a deductive proof on top of an axiom. |  |