 Year 12 Mathematics Standard 1

Unit Title: MS-M3 Right-angled Triangles

Duration 2 weeks

Rationale

Mathematics Standard 1 is designed to help students improve their numeracy by building their confidence and success in making mathematics meaningful. Numeracy is more than being able to operate with numbers. It requires mathematical knowledge and understanding, mathematical problem-solving skills and literacy skills, as well as positive attitudes. When students become numerate they are able to manage a situation or solve a problem in real contexts, such as everyday life, work or further learning. This course offers students the opportunity to prepare for post-school options of employment or further training.

Topic Focus

The principal focus of this subtopic is to solve problems involving right-angled triangles in a range of practical contexts using Pythagoras’ theorem and basic trigonometric ratios.

Students develop their ability to justify mathematical thinking and to communicate solutions.

Within this subtopic, schools have the opportunity to identify areas of Stage 5 content which may need to be reviewed to meet the needs of students.

Prior Knowledge Required

It is assumed that students have completed the Measurement topic in Year 11 Standard course, which covers MS-M1: Applications of Measurement and MS-M2: Working with Time. In particular, they must be able to work with different metric units for length and area, and convert between them, calculate errors in measurement and write measurements to a specified degree of accuracy using significant figures. They should also be fluent in finding perimeters of polygons.

Students have been introduced to the Pythagoras’ theorem in Stage 4 and basic trigonometric ratios in Stage 5.1. However, students are not disadvantaged if there are gaps in their learning from these topics.

Language Considerations

Students may find some of the terminology encountered in word problems involving Pythagoras’ theorem difficult to interpret, for example, ‘foot of a ladder’, ‘inclined’, ‘guy wire’, ‘wire stay’, ‘vertical’, ‘horizontal’. Teachers should provide students with a variety of word problems and explain such terms explicitly.

Teachers must remind students of the correct pronunciation of ‘sin’ as ‘sine’, and encourage students to write the ratios using lower case letters. Many students are taught the acronym ‘SOH-CAH-TOA’ to help remember the trigonometric ratios, but care must be taken to ensure that students do not inadvertently learn ‘SAH-COH-TOA’ due to similar pronunciation. Teachers should clarify that the hypotenuse is not considered as adjacent or opposite.

When expressing fractions in English, the numerator is said first, followed by the denominator. However, in many Asian languages (for example, Chinese, Japanese), the opposite is the case: the denominator is said before the numerator. This may lead to students from such language backgrounds mistakenly using the reciprocal of the intended trigonometric ratio.

Students should be explicitly taught the meaning of the phrases ‘angle of elevation’ and ‘angle of depression’ with the aid of diagrams. While the meaning of ‘angle of elevation’ may be obvious to many students, the meaning of ‘angle of depression’ as the angle through which a person moves (depresses) their eyes from the horizontal line of sight to look downwards at the required point may not be obvious to some students.

Students may find some of the terminology encountered in word problems involving trigonometry difficult to interpret, for example ‘base/foot of the mountain’, ‘directly overhead’, ‘pitch of a roof’, ‘inclination of a ladder’. Teachers should provide students with a variety of word problems and they should explain such terms explicitly with the aid of diagrams.

Students should be able to interpret a variety of phrases involving bearings. They should be taught explicitly how to identify the location from where a bearing is measured and to draw the centre of the compass rose at this location on a diagram. When practising questions that reference a path involving more than one bearing, students may need to be explicitly shown to look for words such as ‘after this’, ‘then’ and ‘changes direction’ that indicate a change of bearing. A new compass rose needs to be centred on the location of each change in direction.

Outcomes

A student:

* MS1-12-3 interprets the results of measurements and calculations and makes judgements about their reasonableness
* MS1-12-4 analyses simple two-dimensional and three-dimensional models to solve practical problems
* MS1-12-9 chooses and uses appropriate technology effectively and recognises appropriate times for such use
* MS1-12-10 uses mathematical argument and reasoning to evaluate conclusions, communicating a position clearly to others

Assessment (including formative and summative)

Some strategies for formative assessment could include:

* Reflecting on students’ responses to a class discussion
* Beginning the lesson with a few questions on content from previous lessons before progressing
* Having students write their own questions on a topic or having them write a specific number of questions with the same answer
* [3-2-1 Exit slips](http://www.theteachertoolkit.com/index.php/tool/3-2-1) - http://www.theteachertoolkit.com/index.php/tool/3-2-1
* [Chalk Talk Routine](http://www.santeesd.net/cms/lib/CA01000468/Centricity/Domain/12/VT_ChalkTalk.pdf) - http://www.santeesd.net/cms/lib/CA01000468/Centricity/Domain/12/VT\_ChalkTalk.pdf
* [Red, Yellow, Green Cups](http://www.sstr2.org/Downloads/Cups%20as%20student%20feedback.pdf) – http://www.sstr2.org/Downloads/Cups%20as%20student%20feedback.pdf. Alternatively, students could indicate beside the questions their level of understanding using the letters, R, Y or G.
* [Mindmaps](https://emedia.rmit.edu.au/learninglab/content/how-create-mind-map) – https://emedia.rmit.edu.au/learninglab/content/how-create-mind-map

Summative Assessment: An assignment titled ‘Mathematics and farming’ that assesses their learning across sub strands in the Year 12 Standard 1 strand, Measurement. A part of the assignment requires students to use Google Earth.

Program

| Content | Teaching and learning strategies and evidence of learning | Resource |
| --- | --- | --- |
| Review the application of Pythagoras’ theorem to solve practical problems in two dimensions AAM | Teachers should begin by verifying students’ understanding of Pythagoras’ theorem. This could be done through a verbal overview of what is Pythagoras’ theorem, with the aid of a diagram of a right-angled triangle on the board. Teachers are encouraged to provide quick questions for students to examine their knowledge of Pythagoras’ theorem.Students should be asked to find the diagonal length of a range of everyday objects that are rectangular prisms using Pythagoras’ theorem, before verifying their answer through measurement, for example, ask the student to measure the diagonal length of their table or the teacher’s table. They should also be given the opportunity to work backwards and investigate the dimensions of a variety of objects, for example. What could be the dimensions of a 65-in (≈164 cm) television?Students should also be given the opportunity to apply Pythagoras’ theorem to everyday objects that are non-rectangular prisms. Examples include finding the minimum length of a straw for a cylindrical cup if the straw is not to be held by hand; finding the minimum height of flower stems when placing flowers in a cylindrical vase; fitting a spoon inside a thermos and then closing it.Students should be given the opportunity to read and practise a variety of Pythagoras’ theorem problems individually and in small groups. Students should be given some word problems to work through as groups and encouraged to decode the problem using a dictogloss (students will need to read a problem and reconstruct it in their own words to demonstrate understanding). They should then find the solution to the problem and share it using strategies such as think-pair-share, justifying their approach to the problem, when there is a disagreement.As food for thought, it is important that the teacher stimulates discussion about scenarios where Pythagoras’ theorem could be used in three dimensions in everyday situations that are meaningful to students. For example, trying to fit long objects into car boot; trying to fit a long log into a wood fired heater. | The following link can be used to generate right-angled triangles to practise Pythagoras’ theorem, with the option of showing fully worked solutions: [Pythagoras’ Theorem Calculator](https://www.geogebra.org/m/Nw7wsSD5), https://www.geogebra.org/m/Nw7wsSD5 Teachers who are interested in using a flipped classroom approach may wish to use the following resource, but they must identify content that is beyond the scope of the syllabus (for example, leaving answers as simplified surds is beyond the scope of this course): [Intro to the Pythagorean theorem](https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-pyth-theorem/v/the-pythagorean-theorem): https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-pyth-theorem/v/the-pythagorean-theorem  |
| Review and extend the use of trigonometric ratios (sin, cos, tan) to solve practical problems AAM* Work with angles correct to the nearest degree and/or minute
 | Teachers should begin by reviewing students’ understanding of trigonometric ratios. They must allocate some time to teaching the following content before delving into practical problems:* Labelling the three sides of a right-angled triangle as opposite, adjacent and hypotenuse with reference to a chosen angle.
* Writing trigonometric ratios for a given right-angled triangle and drawing a right-angled triangle for a given trigonometric ratio.
* Using a calculator to operate with trigonometric ratios.

The teacher should give students an opportunity to practise using the terminology by asking them to complete an information gap activity (see ‘Resources’).The teacher should then reintroduce students to degrees, minutes and seconds as units for measuring angles to varying degrees of accuracy, and teach them rounding angles and calculator usage. Students should practice rounding angles (to the nearest degree and to the nearest minute only) and evaluating expressions involving trigonometric ratios and inverse trigonometric ratios.The teacher should then teach students to find the length of unknown sides of various right-angled triangles using trigonometric ratios. This should increase in difficulty as suggested:* finding unknown sides in the numerator
* finding unknown sides in the denominator
* miscellaneous questions, for example multiple applications of trigonometric ratios

Students should practise a variety of questions to develop fluency.Students should also learn to find the size of unknown angles in right-angled triangles, rounding their answers to the nearest degree and/or to the nearest minute. They must practise to develop fluency.Students must solve a variety of worded problems to apply their knowledge of trigonometric ratios. The teachers should teach students to decode worded problems and to translate it into a labelled diagram.Note: Students should learn to verify if their calculator is in the degrees mode and to change their calculator to degrees mode. Students are not required to use other units such as radians. | Resource 1 – Information gap activity: In pairs, each student will be given a handout with four diagrams and four empty boxes. Students should not show their handout to their partner. They must take it in turns to describe their diagram to the partner to help them draw it exactly. In doing this, they must take into account the orientation of the triangle (For example, the right angle is at the top left corner) and guide their partners to label all the information in their handout.Teachers are encouraged to use calculator emulator software or a poster of the calculator recommended by the school when teaching students to operate with trigonometric ratios.At the end of this section, teachers can extend their capable students with the following challenge that requires them to apply their knowledge of trigonometric ratios and arc length: [Far Horizon](https://nrich.maths.org/2357), https://nrich.maths.org/2357 The following links are ideal for teaching using a flipped classroom approach, but teachers must identify any content that is beyond the scope of the course:[Hypotenuse, opposite, and adjacent](https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-trig-ratios-intro/a/opposite-adjacent-hypotenuse): https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-trig-ratios-intro/a/opposite-adjacent-hypotenuse [Solving for a side in right triangles with trigonometry](https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-solve-for-a-side/a/unknown-side-in-right-triangle-w-trig): https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-solve-for-a-side/a/unknown-side-in-right-triangle-w-trig [Intro to inverse trig functions](https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-solve-for-an-angle/a/inverse-trig-functions-intro): https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-solve-for-an-angle/a/inverse-trig-functions-intro [Right triangle word problem](https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-modeling-with-right-triangles/v/angle-to-aim-to-get-alien): https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-modeling-with-right-triangles/v/angle-to-aim-to-get-alien  |
| Understand various navigational methods* Understand the difference between compass and true bearings
* Investigate navigational methods used by different cultures, including but not limited to those of Aboriginal and Torres Strait Islander Peoples
 | Teachers should begin by ensuring that students know the four cardinal directions (North, East, South, West) and the four intercardinal directions (North West, North East, South West, South East) in the correct order. Teachers should help students to recognise the angles between the directions and demonstrate that the intercardinal locations are located between cardinal directions.Teachers should then teach true bearings as the number of degrees in a clockwise direction from North. Students would benefit from an outdoor learning activity where they measure the true bearings of important locations around the school using a magnetic compass. It may be useful to take a map of the local area and use it to identify the bearings between locations. Students should be allowed to practise a variety of questions where they find the true bearings from A to B and B to A. Students can also measure the true bearing between important landmarks or places of interest using Google Earth. They will need to use the ‘ruler’ tool and look at the ‘heading’ feature (see ‘Resources’ section for guidelines). Students should be allowed to practise a variety of questions, with and without the use of technology (for example, Google Earth). Students should be taught explicitly how to identify the location from where a bearing is measured and to draw the centre of the compass rose at this location on a diagram. They will need assistance in deconstructing worded problems and would benefit from participating in an information gap activity where one student must describe a bearing diagram to another student in order to help them draw it.Teachers should then teach compass bearings as the number of degrees east or west of the north-south line. Students would benefit from practising a variety of questions on the board under step-by-step teacher guidance.Teachers may also wish to discuss other methods of describing direction such as clock positions and port and starboard, which are used in aviation and maritime navigation.Students can identify the use of Aboriginal navigational methods in the past using the article titled ‘How ancient Aboriginal star maps have shaped Australia’s highway network’. The article discusses how the position and pattern of stars were memorised using ‘songlines’ and used to navigate outside of one’s own country. These were also useful to identify ‘waypoints’ such as watering holes. The article highlights the relevance of Aboriginal navigational methods in today’s world since there is similarity between the trade routes and highways when ‘star maps’ are superimposed on modern maps. This is further articulated in three articles (see Songlines articles in ‘Resources’ section).Teachers should then work with students to investigate navigational methods used by at least one other culture (for example, see Resources section for information about Polynesian navigation). Teachers may wish to explore options for cross-curricular learning by collaborating with History, Science and Aboriginal Studies faculties. | Students can also measure the true bearings of important landmarks from another place using the [ruler tool in ‘Google Earth’](https://www.youtube.com/watch?v=x8hzpMn-nNs), as demonstrated in the video: https://www.youtube.com/watch?v=x8hzpMn-nNs[How ancient Aboriginal star maps have shaped Australia’s highway network](https://theconversation.com/how-ancient-aboriginal-star-maps-have-shaped-australias-highway-network-55952)https://theconversation.com/how-ancient-aboriginal-star-maps-have-shaped-australias-highway-network-55952[Songlines and navigation in Wardaman and other Aboriginal cultures](http://www.atnf.csiro.au/people/Ray.Norris/papers/n315.pdf):http://www.atnf.csiro.au/people/Ray.Norris/papers/n315.pdf [Songlines: the Indigenous memory code](http://www.abc.net.au/radionational/programs/allinthemind/songlines-indigenous-memory-code/7581788): http://www.abc.net.au/radionational/programs/allinthemind/songlines-indigenous-memory-code/7581788 [An interactive website on Polynesian navigation](http://annex.exploratorium.edu/neverlost/#/home): http://annex.exploratorium.edu/neverlost/#/home |
| Solve practical problems involving angles of elevation and depression and bearings AAM* Convert between compass and true bearings, for example, convert N35°W into a true bearing
 | It would be beneficial for students to begin the learning of angles of elevation and depression through an outdoor activity involving clinometers (see ‘Resources’ section to make your own clinometer if your school does not own them). They should be asked to measure the angle of elevation and depression of a variety of objects from a partially elevated position. Students should be asked to observe any changes in the angle of elevation and depression as they walk closer to/further away from the base of the object. To help students recognise that angle of elevation equals to the angle of depression, students should be grouped in pairs and asked to identify the bearing of each other from different locations. The findings of the activity should be consolidated in the form of class notes.Students should then participate in another outdoor activity where they measure the angle of elevation and depression of a variety of objects, along with the horizontal distances. Students should be asked to record these in a scaffolded worksheet. Upon returning to class, the teacher should guide students to convert one of the measurements into a diagram and help students to find the length of missing sides in the right-angled triangle and identify what it represents. Students should then be given the opportunity to practise a variety of problems, guided by the teacher when deconstructing complex worded problems. Teachers may wish to explore complex problems involving multiple triangles, including three dimensional problems, to extend capable students. It would be beneficial to construct models using spaghetti and blu tack to help students visualise such complex problems.Once students have developed familiarity with compass and true bearings, they should practise a variety of problems that involves converting a given direction from one form to another. Teachers may wish to extend capable students by giving them harder questions that involve multiple bearings or by teaching students to work with non-right-angled triangles. | [Making Maths: Clinometer](https://nrich.maths.org/5382), https://nrich.maths.org/5382  |

Reflection and Evaluation