 Year 12 Mathematics Advanced

| MA-F2 Graphing techniques | Unit duration |
| --- | --- |
| The topic Functions involves the use of both algebraic and graphical conventions and terminology to describe, interpret and model relationships of and between changing quantities. A knowledge of functions enables students to discover, recognise and generalise connections between algebraic and graphical representations of the same expression and to describe interactions through the use of both dependent and independent variables. The study of functions is important in developing students’ ability to find connections and patterns, to communicate concisely and precisely, to use algebraic techniques and manipulations, to describe and solve problems, and to predict future outcomes in areas such as finance, economics, data analysis, marketing and weather. | 2 weeks |

| Subtopic focus | Outcomes |
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| The principal focus of this subtopic is to become more familiar with key features of graphs of functions, as well as develop an understanding of and use of the effect of basic transformations of these graphs to explain graphical behaviour. Students develop an understanding of transformations from a graphical and algebraic approach, including the use of technology, and thus develop a deeper understanding of the properties of functions. As graphing software becomes more widely accessible, skills in reading scales and interpreting magnification effects become essential. | A student:* uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts MA12-1
* chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9
* constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10
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| Prerequisite knowledge | Assessment strategies |
| --- | --- |
| This topic assumes that students have a sound grasp of the basic functions and algebraic techniques explored in MA-F1. They should also have a sound knowledge of the basic exponential and logarithmic functions studied in MA-E1. | * Formative assessment: Students to use both online graphing tools and pen-and-paper methods to demonstrate informal and, where appropriate, formal investigations and proofs for the concepts explored in this topic.
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All outcomes referred to in this unit come from [Mathematics Advanced](http://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017) Syllabus
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Glossary of terms

| Term | Description |
| --- | --- |
| asymptote | An asymptote is a line.* A horizontal asymptote is a horizontal line whose distance from the function $f(x)$ becomes as small as we please for all large values of$ x$.
* The line $x=a$ is a vertical asymptote if the function $f$ is not defined at $x=a$ and values of $f(x)$ become as large as we please (positive or negative) as $x$ approaches$ a$.
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| break-even point | The break-even point is the point at which income and cost of production are equal. |
| continuous function | A function is continuous when sufficiently small changes in the input result in arbitrarily small changes in the output. Its graph is an unbroken curve. |
| dilation | A dilation stretches or compresses the graph of a function. This could happen either in the$ x$ or$ y$ direction or both. |
| discontinuous | If a function $f(x)$ is not continuous at$ x=a$, then $f(x)$ is said to be discontinuous at$ x=a$. |

| Lesson sequence | Content | Suggested teaching strategies and resources  | Date and initial | Comments, feedback, additional resources used |
| --- | --- | --- | --- | --- |
| Transformations of graphs(3-4 lessons) | * apply transformations to sketch functions of the form 𝑦=𝑘𝑓(𝑎(𝑥+𝑏))+𝑐, where 𝑓(𝑥) is a polynomial, reciprocal, absolute value, exponential or logarithmic function and 𝑎,𝑏,𝑐 and 𝑘 are constants
	+ examine translations and the graphs of 𝑦=𝑓(𝑥)+𝑐 and 𝑦=𝑓(𝑥+𝑏) using technology  Information and communication technology capability icon
	+ examine dilations and the graphs of 𝑦=𝑘𝑓(𝑥) and 𝑦=𝑓(𝑎𝑥) using technology  Information and communication technology capability icon
	+ recognise that the order in which transformations are applied is important in the construction of the resulting function or graph
 | **Note:** Teachers may explore the transformations of all of the functions before teaching the techniques for sketching each of them. Alternatively, teachers may explore the transformations and the techniques for sketching one function at a time. Exploring transformations* Students should explore transformations on graphing software to discover and record how changing particular variables affects each function, with a particular focus on whether it changes the shape or position of the graph with respect to its basic form.
* Model the range of different functions that the transformations can be applied to, including polynomial, hyperbolic, absolute value, exponential or logarithmic functions.
* Use the DESMOS template [Transforming the graph of a quadratic function](https://www.desmos.com/calculator/3tucgd1tiq) to explore$ y=a\left(x-b\right)^{2}+c$. The graph of $y=x^{2}$ is included on the template so that students can describe the comparisons. This template can be easily manipulated to explore the other functions. The DESMOS activity builder [Transforming graphs](https://teacher.desmos.com/activitybuilder/custom/5630e69afa4f9fb51257dcfc) could also provide meaningful explorations for students.
* There should be explicit discussion about how each variable affects each of the different functions in a similar way.

**Sketching graphical transformations*** Explicitly teach the techniques required to graph each function including those skills introduced in MA-F1 such as considering continuity, as well as finding intercepts, asymptotes (vertical and horizontal) and turning points.
* Students will also need to master the algebraic skills to manipulate the expressions and express the functions in the appropriate form to determine the horizontal and vertical shifts.
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| Applying graphical techniques(2 lessons) | * use graphical methods with supporting algebraic working to solve a variety of practical problems involving any of the functions within the scope of this syllabus, in both real-life and abstract contexts **AAM Paperclip icon** Critical and creative thinking icon  Information and communication technology capability icon
	+ select and use an appropriate method to graph a given function, including finding intercepts, considering the sign of $f(x)$ and using symmetry  Information and communication technology capability icon
	+ determine asymptotes and discontinuities where appropriate (vertical and horizontal asymptotes only) Critical and creative thinking icon
	+ determine the number of solutions of an equation by considering appropriate graphs Critical and creative thinking icon
 | Using graphical methods to solve practical problems* Reinforce the idea that a mathematical model is formed from a set of data, finding the rule that links the data together.
* Review the concept of an asymptote and what it means for a graph, as well possible practical implications if the data if being used in modelling.
* It is essential that students are familiar with the shapes and features of all graphs of functions used up to this point. If they are presented with a modelling question, they will first need to recognise the shape of the function that the data creates before finding the rule.
* Some suggestions for situations where models can be applied are:
	+ Linear: distance and time, cost and break-even, capacity and time
	+ Quadratic: maximum and minimum (perimeter vs area), height of a ball
	+ Cubic: maximum and minimum (area versus volume)
	+ Exponential: population growth or decline
* Examples:
	+ A golf ball is launched vertically at 75 meters per second (m/s) from a 20m tall platform. The object's height in metres, t seconds after launch is $h\left(t\right)= –4.9t^{2}+75t+20$. Graph the relationship and determine:

When the ball strikes the ground.The maximum height the ball reaches.When the ball will pass the platform when falling.* + A rectangular paddock is to be fenced using a river as one boundary. The paddock is to have an area of $8000m^{2}$. Let one side of the paddock be $x$ then use a suitable graph to determine the minimum amount of fencing required.
	+ [Mathematics HSC 2015 Q8](http://educationstandards.nsw.edu.au/wps/wcm/connect/33156c3f-4bf4-471a-985a-f3efc2cb0e09/maths-hsc-exam-2015.pdf?MOD=AJPERES&CACHEID=ROOTWORKSPACE-33156c3f-4bf4-471a-985a-f3efc2cb0e09-lGi1XUe) (determine the number of solutions from a graph)
	+ Many past Mathematics HSC questions are algebraic based but sections could be modified or checked with a graphical approach: 2016 Q14c (minimum perimeter), 2016 Q16b (population growth), 2014 Q16c (maximising light), 2010 Q5a (minimum surface area), 2006 Q6b (declining population), 2005 Q6b (volume of water)
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| Solving linear and quadratic inequalities(2 lessons) | * + solve linear and quadratic inequalities by sketching appropriate graphs Critical and creative thinking icon  Information and communication technology capability icon
 | **Solving inequalities by graphical methods*** The introduction of quadratic inequalities in the unit is restricted to solving them based on the graph. As such, the focus should be on correct graphing techniques related to quadratic graphs. From these graphs, solutions can be found.
* Students could plot any linear or quadratic function on DESMOS in the form of $f\left(x\right)=ax^{2}+bx+c$ and write down their observations after using the graphing software to plot $y\geq f\left(x\right)$ and $y<f\left(x\right)$
* The process of solving a quadratic inequality is as follows:
* Factorise the quadratic function
* Draw a quick sketch of the quadratic showing only its $x$-intercepts and concavity
* Highlight the sections of the diagram that meet the given inequality
* Examples:
	+ $2x-7>-3$
	+ $3-8x<2$
	+ $x^{2}+5x+6\leq 0$
	+ $6x^{2}+11x-35>0$
	+ $-x^{2}-5x+18\leq 4$
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Reflection and evaluation

Please include feedback about the engagement of the students and the difficulty of the content included in this section. You may also refer to the sequencing of the lessons and the placement of the topic within the scope and sequence. All ICT, literacy, numeracy and group activities should be recorded in the ‘Comments, feedback, additional resources used’ section.